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SHIFTS IN EARLY NUMBER LEARNING IN SOUTH AFRICA

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In this paper, we report on outcomes from the Structuring Number Starters intervention project focused on early grades’ number learning in South Africa. Using tests drawn from the Maths Recovery programme, a cross-attainment sample of Grade 2 students in six schools took part in oral interview assessments early in 2011, 2014 and 2018, with professional development activities for their teachers occurring across this period. Student outcomes point to increasing proportions, over time, of students able to use more sophisticated counting strategies, and to work with number structure for more efficient calculation.

INTRODUCTION

Low performance in mathematics at all levels of education continues to be a feature of the South African education system. In this context, a public-private partnership was set up in the late 2000s that established a small number of research and development Chairs, based in South African universities, charged with the explicit aim of developing and studying the implementation of research-based interventions to improve learning outcomes across the system. The interventions had to be developed with potential for working at a larger scale if outcomes proved positive. In this paper, we report on the outcomes from an intervention focused on early grades’ number learning – the Structuring Number Starters project – that was run by one of these Chairs within the Wits Maths Connect-Primary (WMC-P) research and development project, and carried out in ten partner schools.

BACKGROUND

Poor performance in mathematics in South Africa has led, over the course of the current decade, to a groundswell of interest in primary mathematics teaching and learning. When the WMC-P project began in 2011 key problems that had been written about in primary mathematics were related to two issues. Firstly, there was wide evidence of primary school students’ approaches to number work being marked by counting based approaches, and often, highly inefficient concrete counting-in-ones or skip counting in multiples (Schollar, 2008). Second was evidence of poor understandings of mathematical progression among primary teachers. One response to this had been the development of a more closely specified curriculum with attempts to highlight the growth of concepts across terms, grades and phases (DBE, 2011), but there is limited evidence that standards have improved as a result of this. Writing across several developing countries also reports on this evidence of low student performance and limited progression, together with the limited success of close curricular prescription (e.g. Pritchett & Beatty, 2012). This evidence indicates that studies of progression in
early numeracy in the South African context are likely to be of broader international interest.

We began our work in South Africa with ten partner primary schools, all serving historically disadvantaged populations. Baseline observations in classrooms across these schools confirmed the prevalence of concrete counting-based approaches to early additive relations problems, which suggested a need for intervention approaches, as described in the international literature, for supporting students to move towards more sophisticated approaches when working with number. In particular, the aims and approaches of the Maths Recovery programme (Wright, Martland & Stafford, 2006) were in line with our goals, and additionally, provided interview-based assessments that we could use to build a profile of the sophistication of strategies Grade 2 students (grade appropriate students aged 6-7) used to answer problems. This was especially useful in a national assessment terrain where tasks were marked as correct or incorrect, without attention to the approaches used to produce answers.

Our broader intervention model was based on working with grade cohorts of students across the school year, focusing on one grade in each academic year. Professional development and intervention activity comprised a termly workshop with WMC-P team members presenting and discussing, with teachers, tasks, activities and resources that focused attention on number structure and relations. The teachers then tried out these tasks, activities and resources with their classes, with one lesson observed and/or co-taught by a WMC-P team member each term. This programme is now in its 8th year. Table 1 summarizes our tracking of cohorts in the years of empirical data collection in the six schools that have worked with the project across this whole period.

<table>
<thead>
<tr>
<th>Year and grade cohort worked with during intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1 (2011-15)</td>
</tr>
<tr>
<td>G2 G3 G1 G2 G3</td>
</tr>
</tbody>
</table>

Table 1: Cohorts tracked across years, assessment data collection years highlighted

As part of our study of the effectiveness of intervention, we administered two parts of the Maths Recovery assessments to a cross-attainment sample of students early in Grade 2 in each of 2011, 2014 and 2018. Grade 2 represents the middle grade of the Foundation Phase in South Africa (Grades 1-3), with some Reception Grade classes coming on stream in the partner schools only in recent years. The broader study of learning has included a written assessment at the start and end of Grade 3. In-depth assessment interviews early in Grade 2 provided us with snapshot data over time of students’ approaches to additive problem-solving, with our intervention work drawing diagnostically from the profiles of performance seen in each of these test administrations. On the research side that we focus on in this paper, this assessment
dataset provides a quasi-longitudinal profile of performance in Grade 2 over time in the context of ongoing intervention activity.

Below, we discuss the literature and theoretical models of progression that underpin these assessments, and then summarise the profile of learning outcomes seen across the quasi-longitudinal test administrations in 2011, 2014 and 2018. We analyse these outcomes, and discuss the implications of shifts in profiles of attainment for policy-making in the broader terrain and for our ongoing work.

LITERATURE & THEORY

In the context of additive enumeration, a broad swathe of international literature clusters around learning trajectories related to increasing efficiency of counting and calculating strategies. One trajectory begins in the terrain of enumerating quantities, drawing on Gelman & Gallistel’s (1986) seminal work setting out the principles that have to be mastered in learning to count (1-1 correspondence, stable order of the number words, cardinality, counts as abstractions, and order irrelevance). Within this trajectory students move into increasingly efficient counting via what can be described as ‘count all’, ‘count on’ or ‘count down from’ strategies, prior to beginning to work with number structure and properties such as commutativity, place value and inverse relationships.

A body of literature questions this trajectory, arguing that instruction should start in number structure and properties set within measurement contexts, rather than start with counting (Davydov, 1990). The argument for this ‘straight for structure’ trajectory is that counting fails to communicate a sense that counts essentially represent a ratio between a unit quantity and the entity being measured by that unit. This, it is argued, creates artificial and unhelpful boundaries between natural numbers and fractions: in the measurement-based approach, rational numbers arise ‘naturally’ alongside whole numbers.

The South African curriculum context largely works with the former – a counting into calculating trajectory – a perspective firmly embedded in the practices of our partner schools. Reflecting this focus on counting, studies of the 2011 project dataset pointed to students getting correct answers to additive questions in lower number ranges merely by using concrete counting strategies rather than through any combination of calculating and reasoning based on number structure (Weitz & Venkat, 2013).

Our intervention work in the Structuring Number Starters project has, however, moved increasingly towards a ‘straight for structure’ model. This has involved the development of task sequences and resources/representations underpinned by attention to numerical relationships. Examples include a focus on using single digit number bonds and base ten related number combinations to develop non-count-by-one calculation strategies, using semi-structured number lines (see https://www.wits.ac.za/wits-maths-connect/wits-maths-connect-primary/structuring-number-starters/term-1-resources/ for further examples). Developing familiarity with
‘five’ and ‘ten’ as useful benchmarks for efficient calculation is a feature that recurs prominently in these tasks.

While moving towards the ‘straight for structure’ approach in our development work, the broader curricular context, and the underlying agreement between the counting into calculating and the straight-for-structure advocates on appreciation of structure as a key goal in early number learning, led to our working with the ‘counting into structure’ trajectory for analysis of counting and calculating efficiency among students. We chose to work with Wright, Martland and Stafford’s (2006) Learning Framework in Number (LFIN) which deals with a range of number aspects (including forward and backward number counting, and numeral recognition), but is centred around a spine in which strategic counting efficiency leads into further efficiencies involved in working with number relationships. Wright and colleagues describe the hierarchies in this spine as the ‘Stages of Early Arithmetic Learning’ (SEAL), with the label ‘stages’ marking what they see as the substantial conceptual reorganizations involved in the moves between steps. These stages, in paraphrased terms, are outlined in Table 2.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Title</th>
<th>Description of counting/calculating strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Emergent count</td>
<td>Cannot count a visible collection of counters</td>
</tr>
<tr>
<td>1</td>
<td>Perceptual count</td>
<td>Can count perceived items (seen, heard or felt) and solve additive tasks involving displayed collections</td>
</tr>
<tr>
<td>2</td>
<td>Figurative count</td>
<td>Can solve additive tasks involving one or two screened quantities using the ‘count all’ strategy</td>
</tr>
<tr>
<td>3</td>
<td>Initial number sequence</td>
<td>Can solve addition tasks using ‘count on’, ‘count-up-to’ and ‘count-down-from’</td>
</tr>
<tr>
<td>4</td>
<td>Intermediate number sequence</td>
<td>Can use ‘count-down-to’ to solve missing subtrahend tasks and can choose the more efficient of ‘count-down-from’ and ‘count-down-to’ for task at hand</td>
</tr>
<tr>
<td>5</td>
<td>Facile number sequence</td>
<td>Can use a range of ‘non-count-by-one’ strategies involving calculation-by-structuring (doubles, near doubles, compensation, 1010, N10 and bridging-through-ten) and known and derived facts</td>
</tr>
</tbody>
</table>

Table 2: The Stages of Early Arithmetical Learning

In Stages 1 and 2, the enumeration of quantities relies on unit counting of all the quantities presented in the problem – each part, and then the whole, have to be counted out from one. ‘Count on’ and ‘count down from’ strategies at Stage 3 shift this triple count to a single count – on or down from the starting number. Hence the shift from Stage 2 to Stage 3 in Wright et al.’s model is important in that it marks the first move to any reification of counting processes into number objects (Gray, 2008). Whilst Stage 3 represents an important shift in efficiency terms, it remains, as van den Heuvel-Panhuizen (2008) notes, within the terrain of ‘calculating-by-counting’ strategies,
rather than the ‘calculating-by-structuring’ strategies which involve both attending to relationships between numerical quantities and also deriving efficient strategies based on number fluencies that present themselves as recalled facts. It is in Stage 5 that such calculation by structuring, using number relations and properties, comes to the fore, with Stage 4 laying the groundwork for this move.

**METHODOLOGY**

Each of our partner schools were asked to select six Grade 2 students from across the mathematics attainment range: two under-attaining students, two average attaining students and two higher-attaining students. Following the Math Recovery model, in-depth 1-1 assessment interviews were conducted with each of the selected students (with informed consents from the children, their parents/carers, teachers and school principals). The interviews, conducted with student first language translators where needed, were video-taped in order to capture gesture-based counting strategies (in particular, finger counting) which, as we noted earlier, were widely prevalent.

Subsequent coding of each child’s assessment responses using the LFIN led to the development of individual learner profiles. In this paper, we focus on the SEAL aspect only, described in Wright et al’s work as the single most important aspect in relation to supporting early number progression in the overall LFIN profile. Although we worked with ten partner schools each year, six schools were involved across each of the 2011, 2014 and 2018 test administrations, so we focus here on the profile of SEAL stages seen across the 108 students interviewed: 36 in each year.

**FINDINGS AND ANALYSIS**

In Table 3, we summarize the SEAL Stage profiles across 2011, 2014 and 2018 of the cross-attainment sample of 36 learners from the six schools participating across all three test sittings.

<table>
<thead>
<tr>
<th>SEAL Stage</th>
<th>2011</th>
<th>2014</th>
<th>2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4 (11.1%)</td>
<td>1 (2.8%)</td>
<td>1 (2.8%)</td>
</tr>
<tr>
<td>1</td>
<td>8 (22.2%)</td>
<td>4 (11.1%)</td>
<td>1 (2.8%)</td>
</tr>
<tr>
<td>2</td>
<td>15 (41.7%)</td>
<td>9 (25%)</td>
<td>10 (27.8%)</td>
</tr>
<tr>
<td>3</td>
<td>8 (22.2%)</td>
<td>20 (55.6%)</td>
<td>15 (41.7%)</td>
</tr>
<tr>
<td>4</td>
<td>1 (2.8%)</td>
<td>2 (5.6%)</td>
<td>7 (19.4%)</td>
</tr>
<tr>
<td>5</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>2 (5.6%)</td>
</tr>
</tbody>
</table>

Table 3: SEAL profile across 2011, 2014 and 2018 (n=36 in each year)

We considered the data quantitatively and more qualitatively to understand SEAL profiles over time. Drawing from the literature noting that the move from Stage 2 to 3 marks an important transition into beginning to see numbers as reified structures, we
carried out a chi-square test of independence to examine the relation between grouped SEAL stages 0-2 and 3-5 and the year of assessment. This grouping ensured that all cells contained five or more students, allowing for a valid chi-square test application. The null hypothesis here was that the distribution of students across these two combined levels was independent of the year of assessment. Contradicting this hypothesis, the relationship between these variables was highly statistically significant: \( \chi^2 (2, N = 108) = 14.75, p < .001 \), indicating that differences between the years were highly unlikely to be due to chance alone.

Returning qualitatively to the individual SEAL stages, a graphical representation of the proportions of students using the strategies associated with each of the SEAL Stages illustrates the shifts in the profile of strategy use over time (see Figure 1).

![Figure 1: SEAL profile over time](image)

Several features of the shift in profile over time illustrated in Figure 1 are noteworthy. Most importantly, in 2011, 75% of the student cohort in our cross-attainment range sample were assessed as using the concrete unit counting strategies associated with Stages 0-2, but by 2014, this proportion had dropped to 38.9% of the student cohort, and by 2018, had decreased further to 33.4% of the student cohort. The key shift seen in the 2014 attainment profile is a large increase in the students assessed as being at Stage 3 – the ‘count on’/’count down from’ stage – which, as we noted earlier, marks the start of reifications of counting processes into number objects. There is also a shift in 2014 to over 60% of the students exhibiting strategies related to SEAL Stages 3 to 5, in contrast to only 25% of the student cohort being in this range in 2011.

The key shift in 2018 is evidence of growing proportions of students able to work with number structure and relationships in ways that move them beyond counting-in-ones strategies. In 2018, 25% of the cross-attainment sample used SEAL Stages 4 & 5 in their responses – the stages in which ‘calculating-by-structuring’ comes into play making use of relationships between numbers for increasingly efficient working. This proportion is substantially higher than the 2.8% of the cohort at these stages in 2011 and the 5.6% at these stages in 2014. This large increase points to successes associated
with teachers’ work with tasks underpinned by the ‘straight for structure’ approach in the Structuring Number Starters project.

CONCLUDING COMMENTS

These shifts in the profiles of student performance across the years point to teaching and learning that is moving forwards in relation to strategic efficiency and a stronger sense of number structure and relationships. Such a move is important given the literature showing that this is necessary for subsequent working in expanded number ranges and for algebraic thinking (Kieran, 1996). There is also though, further work to be done with the stubborn third of students who remain in the concrete counting from one stages, and the roughly 40% of the sample working with ‘count-on’ and ‘count-down-from’ strategies at Stage 3 in 2018. This work goes hand-in-hand with simultaneously working further moves into the more efficient relational strategies associated with SEAL Stages 4 and 5.

This analysis provides us with useful insights into changes over time in strategic efficiency, in the context of our broader development activity. The terrain of our work was one in which multiple initiatives were at play – national curriculum changes, the rollout of national workbooks, and provincial initiatives focused on supporting curriculum coverage and pacing through the provision of scripted lesson plans. What demarcated our intervention from these broader issues was specific attention to instruction focused on number structure and strategic efficiency, rather than more generic attention to curricular coverage. Additionally, our move into a ‘straight-for-structure’ approach appears significant in its association with the substantial increase in the proportions of students now displaying approaches in the upper, structural, SEAL stages. The analysis suggests that the Structuring Number Starters project is paying dividends in supporting both pedagogic attention and also learning outcomes towards efficient working with number based in a sense of number structure and relationships. As such, these findings are of broader interest given the evidence from contexts of disadvantage and poverty in several parts of the world where many students’ poor performance in number in the early grades remains a live concern.

Acknowledgments

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References


