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Concentration Risk in the HDAX and its Influence on the Index Variance

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Abstract

The aim of this paper is to provide a comprehensive analysis on the concentration of the HDAX index and its influence on the index volatility. The HDAX, composed out of the 80 largest German companies as well as the 30 largest technology companies can be seen as a mirror of the German economy. To understand the influence of single companies on the index we conducted a detailed analysis checking for concentration and variance. This paper measures the effects on the index by applying daily return data of the companies included in the HDAX from 1998 until 2014. After recreating the HDAX, concentration ratios and deciles are formed to get an insight of the weight composition within the index. The Gini coefficient and the Lorenz curve are used to investigate for inequality in the composition. The outcome shows that the HDAX is highly concentrated with the top ten companies making up on average 60% of the index weighting. Furthermore, a Garch(1, 1) Model is applied to observe if high concentration during the years of 2007 and 2008 implies higher volatility. Additionally, we simulated the weights of each index constituent with low, intermediate and high level of concentrations to test for its influence on the variance. No direct correlation between the concentration and the variance was found with either method. For verification measures, we created minimum variance portfolios with the constituents of the HDAX to control if the concentration of the index composition differs significantly, while the variance is minimised. Likewise, the results confirm no relationship between concentration in the HDAX and the observed volatility.

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1 Introduction

1.1 General

An issue of contemporary concern in many national stock exchanges is market concentration. Especially in the UK, a discussion started in year 2000 about the fact that multi-national companies have grown so large over the past years that they dominate the national capital-weighted indices (Chelley-Steeley, 2008). This so called 'concentration risk' within an index or market describes the circumstance that large companies are growing in value at a disproportionate rate compared to their smaller counterparts. This increases their influence on the performance of the index. Figure 1 shows that there is a large disparity between the concentrations in terms of market capitalisation

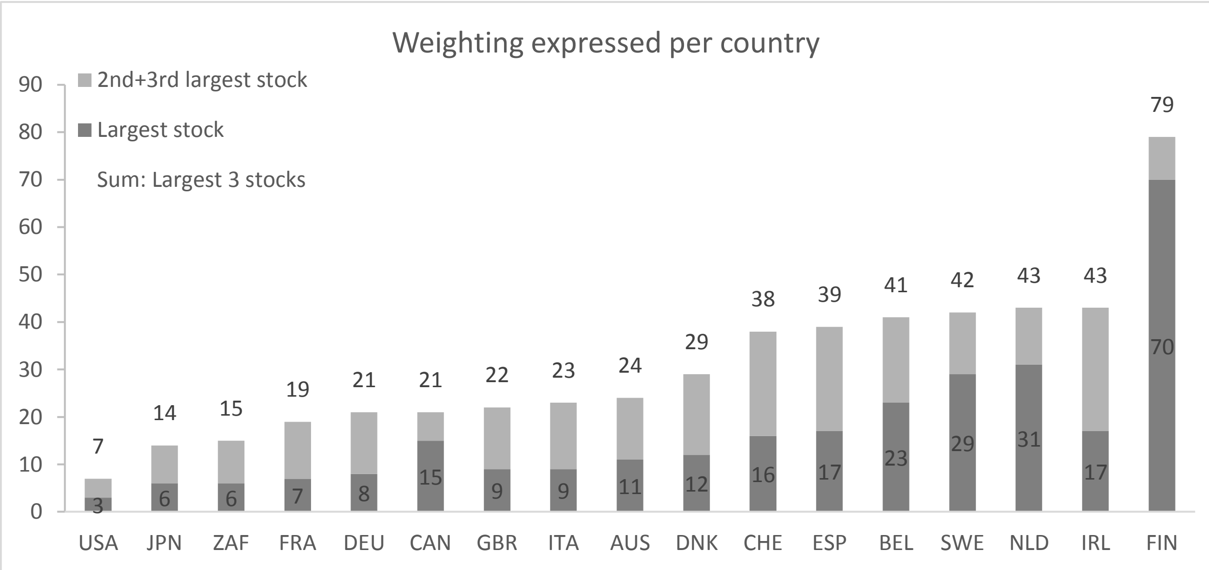

Figure 1: Weighting expressed as % of total country market capitalisation as of start 2001. Source: (Dimson et al., 2009, P.29.)

in different countries. It illustrates that in Germany in the year of 2001 the market capitalisation of the three largest companies made up 21% of the total German stock market capitalisation. With that, Germany is located in the lower midfield of the regarded countries. What the graphic does not show, are the effects that a high concentration has on the corresponding national indices. The aim of this paper is to check for concentration within the German HDAX and the influence it might have on the risk of the index, measured by the volatility and variance as key figures. The research question we are trying to solve is if a high concentration, meaning that only a few companies make up a large proportion of the capital-weighted index, has a positive, negative or no effect on the variance of the index. For that purpose, we implemented the methods used by Chelley-Steeley (2008) to analyse the concentration in the German stock market. We will consider not only the three largest companies as shown in Figure 1, but instead analyse the weight distribution of the 80 largest German companies as well as the 30 largest German technology companies over the time period of 2003 to 2014 to create a complete picture on the concentration and its effect on the German market.

Section two will give a short overview over the conducted research on this topic. It is followed by a short description of the usage of indices in the financial market. Section four will deal with the theoretical framework that has been used in this paper, which is followed by a detailed explanation of the used data as well as the methods that were applied to conduct our research. Section six describes in detail the corresponding results of the research. The final segment gives an overall conclusion on the research, discusses the findings critically and gives suggestions for future research.

1.2 Practical applications

Stock market indices are mostly used for three purposes, first as a benchmark against which the performance of the manager is measured, second as a way of investing in the market and third as hedging instrument.

The first one is to measure how well a portfolio manager has performed by comparing the difference between the fund manager's portfolio return and the one of the according benchmark. Over the past decades, a substantial trend can be observed toward making the compensation of portfolio managers dependent on their performance relative to a certain benchmark index (Admati and Pfleiderer, 1997). Brennan (1993) comes up with the assumption that the manager's reimbursement is based on performance interrelated to an index. He studies the equilibrium that results when a considerable part of the market is invested by participants responding to such contracts. By evaluating the portfolio holdings of the mutual funds, it is necessary to design benchmarks, which capture the investment style (Kent, Grinblatt, Titman and Wermers, 1997).

Second, stock indices futures can be used as an investment instrument by building exposure to a market or sector without purchasing shares directly. In 1982, when stock index futures had their first appearance investors widened their investment range which enabled the portfolio managers to enhance their risk management strategy (Chiu, Wu, Chen and Cheng, 2005).

As a last instrument stock market index futures can be used for hedging against a portfolio of shares or equity index options. Assumed that the hedging will be done perfectly, the investor will not make any gains on the index, rather the investment will only gain the amount equal to the risk free rate. Hence, the hedge ratio uses the benefits generated in the futures market to compensate for potential losses obtained in the spot market (Chiu et al., 2005). When the portfolio does not entirely mirror the index, which is more common, the degree of correlation between the hedge and the underlying assets is rather small.

Another way to use index futures is by leveraging the portfolio and increasing the exposure to the movement of the index. Furthermore, stock index futures can be used for volatility trading where the objective is to use the difference between the implied volatility of an option and the forecast of future realised volatility of the options underlying (the higher the volatility the greater the probability of profit taking). In volatility trading (also called volatility arbitrage) not the price is used as a measurement for the unit, but the volatility.

2 Literature review

Not much research has been conducted on the topic of concentration risk within an index. One of the few studies that addresses the concentration within a stock market is by Dimson, Marsh and Staunton (2009). They study the market concentration of 16 countries, which account for 88% of today's equity market and 95% of the world's bond market from 1900 until 2001. They are able to show that stock market concentration varies greatly across the different countries and fluctuated considerably over time. Furthermore, they show that even though the concentration ratio in the UK, which has been discussed especially around 2001 and 2002, is lower than the average. The pioneer, and so far the only study that addresses the topic of concentration risk in an index is conducted by Chelley-Steeley (2008). She analyses the effect of concentration on the volatility of the British FTSE 100 Index. Especially the years of 2000 to 2002 are regarded closely. Her findings state that the level of concentration has no effect on the volatility and variance of the index. Instead, a lower concentration might sometimes have a bad effect on the volatility.

It can be assumed that the companies accounting for a high concentration in the index are multinational companies (MNC) that have business activities around the world. Agmon and Lessard (1977) find, that companies that are globally active usually have a more diversified risk and therefore their stock prices are less volatile. Mathur and Hanagan (1983) as well as Hughes, Logue and Sweeney (1975) correspondingly conclude that MNC face less risk due to their greater size and because of their stable earnings resulting out of their international diversification. Hughes et al. (1975) also checks for systematic and unsystematic risk between MNC and domestic companies. They conclude that multinational firms have a lower systematic as well as a lower unsystematic risk than domestic companies. Hence, a high concentration could lead to lower volatility. Since only little research has been conducted, we use concentration measures that have been used greatly in industrial economies to describe market power. Hannah and Kay (1977) define the process of concentration as "an increase in the extent to which economic activity is controlled by large firms". This definition can also be used for an index concentration with the modification that in the case of an index, not the economic but the index activities are controlled by large firms.

3 Theoretical framework

This section discusses the most important theories regarding the economic and statistical methods applied. Following, we explain the calculation of the composition of the HDAX, as well as two measurements of inequality, the Gini coefficient and the Lorenz curve. At last, theoretical explanations regarding the minimum variance portfolio are given.

3.1 Risk measurement

Covariance

In the theory of probability, covariance is a measure of how two random variables change or vary together. It is used to measure the linear relationship between two random variables. The covariance can either have a positive or negative value. There is a positive covariance if the value of one variable corresponds positively with the value from a different variable. If the value of one variable moves in the opposite direction than the value of the other variable, the covariance tends to be negative. The normalised version of the covariance, also called correlation coefficient, however, shows by its magnitude the power of the linear relation between two variables. The covariance between two jointly distributed real-valued variables X and Y is defined in equation (1):

$$Cov(X, Y) = \sigma(X, Y) = E[(X - E(X))(Y - E(Y))] \quad (1)$$

Equation 1: Covariance

where $E(X)$ and $E(Y)$ are the respective means. For the covariance between stocks, the model can be rewritten as shown in equation (2):

$$Cov(ABC, XYZ) = \frac{\sum_{i=1}^{N}((Return_{ABC,i} - Average_{ABC}) * (Return_{XYZ,i} - Average_{XYZ}))}{N - 1} \quad (2)$$

Equation 2: Modified Covariance formula

where the $Return_{ABC,i}$ is the log return of the time period i. The covariance is an unstandardised version of the correlation. To compute the corresponding correlation (can take on values between -1 and 1) the covariance is divided by the standard deviation of both variables to remove units of measurement.

Variance-Covariance Matrix

The variance-covariance matrix is a square symmetric matrix in which the elements on the main diagonal are variances and the remaining elements are covariances. Suppose X_1, X_2,…, X_p are random variables and the variance of X_j is $\sigma_{j,j}$, additionally the covariance of X_j and X_k is $\sigma_{j,k}$=$\sigma_{k,j}$.

Then the variance–covariance matrix is given by equation (3):

$$\Sigma = \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \sigma_{1,3} & \cdots & \sigma_{1,110} \\ \sigma_{2,1} & \sigma_{2,2} & \sigma_{2,3} & & \sigma_{2,110} \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_{3,3} & \cdots & \sigma_{3,110} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \sigma_{110,1} & \sigma_{110,2} & \sigma_{110,3} & & \sigma_{110,110} \end{pmatrix} \quad (3)$$

Equation 3: Variance-Covariance Matrix

The covariance matrix generalises the notion of the variance to multiple dimensions.

3.2 Minimum Variance Portfolio

Markowitz (1952) identified in his Nobel prize winning theory the trade-off that investors face in all their economic decisions: risk versus expected return. In his work, he tried to identify all possible portfolios that minimise risk, which is measured as standard deviation for a given level of expected return. To illustrate this graphically as shown in Figure 2, the portfolios located on the efficient frontier, represent the trade-off between risk and expected return an investor faces when compiling an efficient portfolio. These combinations show well-diversified portfolios, which achieve a risk reduction. The minimum variance portfolio is the point on the efficient frontier where the standard deviation is being minimised.

The optimal portfolio selection process is determined through portfolio weights, which diversify securities in the most efficient way by having the lowest possible level of volatility.

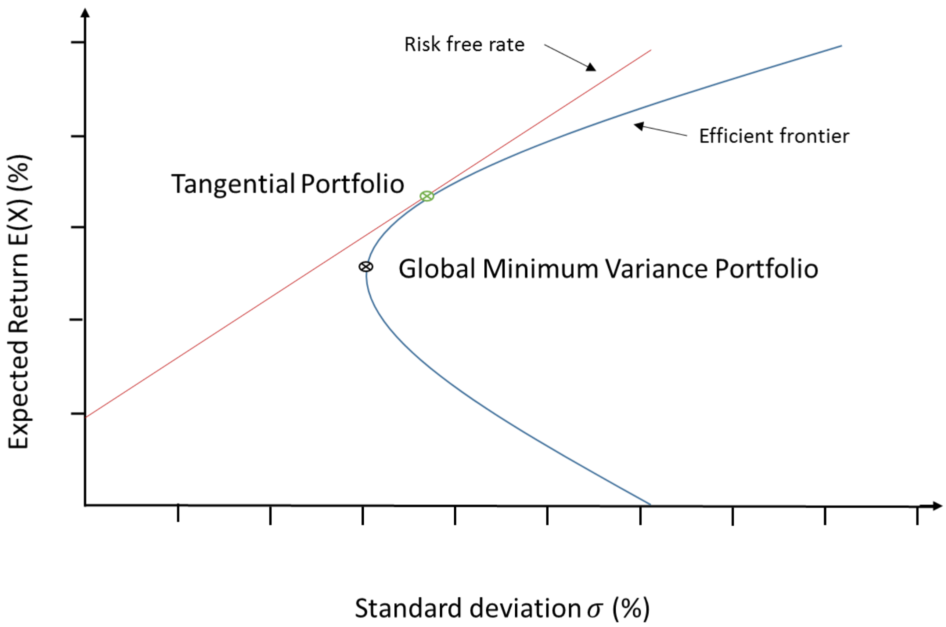

Figure 2: Minimum Variance Portfolio

Overall, Markowitz developed a mean-variance analysis by selecting the optimal portfolio of common stocks relying on an efficient combination of the variance/covariance of the constituents. However, a considerable risk within the estimation of the covariance matrix remains (Kempf and Memmel, 2003). Furthermore, it is shown in several former studies (see Haugen and Baker (1991), Clarke, Silva and Thorley (2006), Poullaouec (2008)) that minimum variance portfolios have reported similar returns like their benchmark indices, but with an on average 25% to 30% lower standard deviation. For this reason, minimum variance portfolios gained popularity and are widely used as an optimisation tool for a number of investment funds since 2007 (Keefe, 2008; Appel, 2008).

3.3 HDAX

The HDAX is a capital-weighted index published by Deutsche Börse AG. It was first established in 1994 as the DAX100. After the recomposition of the index in 2003, the HDAX aggregated the 80 largest German companies listed in the Prime Standard Segment of the Frankfurt Stock Exchange, composed out of the 30 largest companies in the DAX and 50 companies in the MDAX as well as the 30 largest technology companies listed in the TecDAX (Deutsche Börse, 2003). It represents the largest German blue chip index that covers all sectors listed in the Prime Standard Segment. The index is capital-weighted, whereby every individual company is represented proportionally to the overall capitalisation of all the participants in the HDAX. The weighting of each issue is based on the free float portion of the issued shares times the market value (free float in % x market value).

The HDAX is being reviewed quarterly and possible adaptions to the index are being made in accordance to the ground rules published by Deutsche Börse. During the quarterly chaining process, the free float factor as well as the number of shares issued are updated and a new market value is being calculated. According to the newly computed weights, the index is built, among others, on the following general rules:

- Company needs to be listed in Prime Standard
- Is traded on Xetra continuously
- Minimum free float of 10%
- A company cannot amount for more than 10% of the whole index

A more detailed explanation on the construction of the HDAX index can be found in the Methodology section.

If the general rules are fulfilled, the selection of the index constituents for the HDAX is based on the following criteria:

- Order-book sales in Xetra and on the (trading) floor in Frankfurt
- Free-float capitalisation at the last trading day in the month

The change of index constituents is announced on the evening when the decision is made and thus, in time before the new chaining through Deutsche Börse AG. The calculation of the index is shown in equation (4):

$$HDAX_t = K_{t1} * \frac{\sum_{i=1}^{110}(p_{it}*ff_{it1}*q_{it1}*c_{it})}{\sum_{i=1}^{110}(p_{i0}*q_{i0})} * 500 \quad (4)$$

Equation 4: Index formula

where

t_0 is 30.12.1987

t_1 is the chaining day

p_{it} is the current price of the company i

ff_{it1} is the free float of the class i on the chaining date

q_{it1} is the number of shares of company i on the chaining date

p_{i0} is the price of company i on 30.12.1987

c_{it} is the current adjustment factor

K_{t1} is the chaining factor

Through extension of the basic formula with $\frac{100}{\sum_{i=1}^{110} q_{i0}}$ and converting at the same time, we get equation (5):

$$HDAX_t = \frac{\sum_{i=1}^{110}\left(p_{it}*\left[ff_{it1}*q_{it1}*c_{it}*K_T*\frac{100}{\sum_{i=1}^{110} q_{i0}}\right]\right)}{\sum_{i=1}^{110}\left(p_{i0}*q_{i0}*\frac{100}{\sum_{i=1}^{110} q_{i0}}\right)} * 500 \quad (5)$$

$$= \frac{\sum_{i=1}^{110}(p_{it} * F_{it})}{A} * 500$$

Equation 5: Adapted index formula

where

- The weighting according to the index formula is the number of freely available shares $ff_{it1} * q_{it1}$.
- The expression A is a constant value, which changes only, when the composition of the index is updated.
- Therefore, a recalculation of the index is possible in three steps:
    1. The current prices are multiplied with F_{it}
    2. The determined prices of 1. are summed up and divided by A
    3. The result is multiplied with the base of 500
- The current percentage G_{it} of a company in the HDAX corresponds to the share of the current market value of the company on the sum of all current market values. G_{it} shows the influence of one constituent on the index. The calculation is shown in equation (6):

$$G_{it} = \frac{q_{it} * p_{it} * ff_{it1} * c_{it}}{\sum_{i=1}^{110}(q_{it1} * p_{it} * c_{it})} * 100 = \frac{p_{it} * F_{it}}{\sum_{i=1}^{110}(p_{it} * F_{it})} * 100 \quad (6)$$

Equation 6: Company weighting within the HDAX (Deutsche Börse, 2014)

3.4 Gini coefficient & Lorenz curve

The Gini coefficient is established as a conventional measure of inequality. The coefficient can vary between one and zero, where one stands for the highest degree of inequality, while zero stands for an equal distribution. There are different approaches published on how to calculate the Gini coefficient (see Atkinson (1970), Sen (1973), Fei, Ranis and Kuo (1978), Gini (1912), Theil (1967)). The Gini coefficient can be regarded as an important statistic on the Lorenz curve. The Lorenz curve is in our case the proportion of the total concentration of the index constituents, distributed and starting with the smallest company. The Lorenz curve plots the cumulative company weights, starting with the smallest, against the cumulative proportion of firms. If each company has the same weight, the Lorenz curve would be a 45° line, however, the more concave the line is, the more unequal the weights and the higher the concentration of the index. Taking this into account, the Gini coefficient is the area between the Lorenz curve of a concentration at time t and the Lorenz curve for an index in which every company is equally weighted (45° line). This is expressed as a proportion of the area under the curve between the equal distribution and the Lorenz curve at time t (Dorfman, 1979). For illustration see Figure 5.

4 Data and methodology

The following section gives an inside on the data collection process as well as on the execution of the study. In the latter part different types of statistical methods that were used during the process of our research are presented.

4.1 Data

We use the German HDAX index as a basis for our concentration risk measure. The index consists of 110 constituents and was introduced in March 2003. The index comprises of the 80 largest German companies as well as the 30 largest technology companies in Germany. As data we use daily stock prices from 1998 to 2014 (in Euro) traded on XETRA, the market value (in millions of Euro) and free float (in percentage of outstanding shares) of the companies. The data of the study consists of 279 companies that were included in the HDAX, at one point over the observed period. Furthermore, we use daily index data for the HDAX. The data was extracted from Thomson Reuters Datastream.

4.2 Methodology

4.2.1 Concentration

As a first measure of concentration, we create ratios existing out of one to ten companies with the highest weights to get a first impression about the distribution in the index. The concentration ratios are calculated through equation (7):

$$C_n \sum_{n=1}^{n} \omega_i \quad n = 1 \ldots 10 \tag{7}$$

Equation 7: Concentration ratio

It compromises the aggregate weighting of the n largest firms in the index, where C_n is the concentration ratio itself. The market weighting of each security i is shown as ω_i.

4.2.2 Hirschman-Herfindahl Index

Since there are different concentration measurements, another way of checking the level of concentration is by controlling for the distribution of firm sizes. The Hirschman-Herfindahl Index (or H ratio) estimates the concentration by focusing on the dispersion of companies within the index and is estimated according to equation (8):

$$H_n \sum_{n=1}^{n} (\omega)^2 \tag{8}$$

Equation 8: Hirschman-Herfindahl Index

where $(\omega_i)^2$ is characterised by the squared company weights. In our case $(\omega_i)^2$ are the squared market weights of each security constituent of the HDAX index. Compared to the concentration ratios, the H ratio utilises information of all 110 companies in the index. The value of the H ratio lies at $1/110$, if all companies have equal weights, however increases with rising concentration up to 1. Theoretically this would be the case if the measured index was made up of only one company.

4.2.3 Index variance

In this section we address the connection between index volatility and concentration. This is done by comparing if concentration changes within the HDAX are directly correlated to volatility changes. At the end we observe the variances and covariances of each HDAX constituent to explain possible changes in the index.

As already mentioned, the index variance is determined by the weighted variances and covariances of the component securities as illustrated in equation (9):

$$Var(I_t) = \sum_{i=1}^{110} X_i^2 \sigma_i^2 + \sum_{i=1}^{110} \sum_{\substack{j=1 \\ j \neq i}}^{110} X_i X_j \sigma_{i,j} \qquad (9)$$

Equation 9: Portfolio variance

It shows that it is not possible to determine the effect of concentration on the index variance only by studying the variances; rather the variances as well as the covariances of each component security need to be taken into account. The $Var(I_t)$ is the variance of the stock market index at time t. X_i are the weights according to each component security of the HDAX, σ_i^2 is the variance of each security i's return and $\sigma_{i,j}$ shows the covariance between company i and j.

The variance of the 110 participants of the HDAX index is multiplied with the corresponding squared company weighting. Furthermore, there are 11,990 covariance terms for each point in time, which are successively weighted by the product of the weighting of each stock i and stock j. The covariances are calculated using Excel as well as its add-in "Analysis ToolPak".

In addition, equation (9) shows that if either the variance or the covariance of constituent securities will increase, the variance of a portfolio will increase as well. Compared to this, changes in the weighting structure can have different impacts on the index volatility. An increase in the weight of constituents with lower covariances as well as lower variances will rather reduce the index variance. Nevertheless, the effect of lowering the weighted variances (covariances) could be counterbalanced if firms that obtain a higher weighting also have higher variances (covariances).

4.2.4 GARCH(1, 1) Model

As a first step we estimate a GARCH(1, 1) Model as shown in equation (10):

$$R_t = \alpha_0 + u_t$$

$$h_t = \beta_0 + \delta_0 D_t + \beta_1 u_{t-1}^2 + \beta_2 h_{t-1} \qquad (10)$$

$$\beta_0 > 0 \qquad u_t \sim N(0, h_t)$$

Equation 10: GARCH(1, 1) Model

The model is estimated for a time period between March 2003 and December 2014. Additionally, we included a dummy variable for the years 2007 and 2008, which are – according to our concentration ratios and deciles – the years with the highest index concentration. The dummy variable has a value of one for the years 2007 and 2008, otherwise zero. If the dummy coefficient will take on a positive value, it would imply that during the period of high concentration, volatility was positively affected. A negative coefficient would suggest a lower volatility during 2007 and 2008.

In equation (10) R_t is the logarithmic return of the HDAX in time t including dividends, since it is a performance index, meaning that dividends are reinvested into the index. The u_t are residuals, h_t is the conditional variance of the HDAX, α, β and δ are coefficients, whereby δ mirrors the coefficient of the dummy variable. For the GARCH(1, 1) Model EViews was applied.

4.2.5 Simulation of weights

In this part of the paper, we investigate the sensitivity of the HDAX index according to different levels of concentration by simulating the weights of each year using a level of low, intermediate and high concentration. At the end of each year, we fix the constituents of the HDAX and assign to each of the securities the variances and covariances that were previously computed. Based on this, we create a 110 x 110 variance-covariance matrix. To each diagonal element we add the squared weight and to each non-diagonal element we add the product of both weights as shown in matrix (11):

$$\begin{pmatrix} X_{1,c}^2\sigma_{1,1} & X_{1,c}X_{2,c}\sigma_{1,2} & X_{1,c}X_{3,c}\sigma_{1,3} & \cdots & X_{1,c}X_{110,c}\sigma_{1,110} \\ X_{2,c}X_{1,c}\sigma_{2,1} & X_{2,c}^2\sigma_{2,2} & X_{2,c}X_{3,c}\sigma_{2,3} & \cdots & X_{2,c}X_{110,c}\sigma_{2,110} \\ X_{3,c}X_{1,c}\sigma_{3,1} & X_{3,c}X_{2,c}\sigma_{3,2} & X_{3,c}^2\sigma_{3,3} & \cdots & X_{3,c}X_{110,c}\sigma_{3,110} \\ \vdots & & & & \vdots \\ X_{110,c}X_{1,c}\sigma_{110,1} & X_{110,c}X_{2,c}\sigma_{110,2} & X_{110,c}X_{3,c}\sigma_{110,3} & \cdots & X_{110,c}^2\sigma_{110,110} \end{pmatrix} \qquad (11)$$

$$c = 1,2,3$$

Equation 11: Variance-Covariance Matrix including simulated weights

By using the variance-covariance matrix, adjusted by the weights, we calculate the variance of the synthetic HDAX index with equation (12):

$$Var(I_t) = \sum_{i=1}^{110} X_{i,c}^2 \hat{\sigma}_{i,t}^2 \sum_{\substack{i=1 \\ j\neq i}}^{110} X_{i,c}X_{j,c}\sigma_{i,j,t} \qquad (12)$$

$$i = 1, \dots, 110 \quad j = 1, \dots, 110 \quad i \neq j \quad c = 1, 2, 3$$

Equation 12: Variance of synthetic index

$Var(I_t)$ is the variance of the HDAX index including all 110 companies during the regarded five year period t, $\hat{\sigma}_{it}^2$ is the estimated variance of each index constituent i for the according sub-period t, which is estimated by using monthly returns from the previous five years. The weights corresponding to each constituent are shown by $X_{i,c}$ and $X_{j,c}$. The c stands for the three different weightings used for the simulation of the index.

The weights are fixed on a yearly basis at the end of December to calculate the index variance and can take on three different values, namely low, intermediate and high concentration. The year of 2008 is chosen to reflect high concentration, 2006 reflects intermediate and 2014 low concentration, respectively. Based on this, we calculate the variance of the HDAX during a five-year period by using all three levels of concentration. To compute the different synthetic indices we use the Excel Add-in Solver and implemented 110 restrictions for each of the three concentration level: that is each weighting in the future portfolio (index) must coincide with the weighting of the chosen concentration level (low, intermediate, high).

4.2.6 Minimum Variance Portfolio

The minimum variance portfolio is used to study the link between the volatility and the concentration characteristics of the HDAX by minimising the variance of the portfolio according to equation (13):

$$Min \quad \sum_{i=1}^{110}\sum_{j=1}^{110} X_i X_j \sigma_{i,j}$$

$$subject\ to\ X_i \geq 0, \quad i = 1, 2, \ldots, 110 \tag{13}$$

$$\sum_{i=1}^{110} X_i = 1.0$$

Equation 13: Minimum Variance Portfolio

The minimum variance portfolios are calculated based on the index composition at the end of each year using the five-year backward period of variances and covariances. When calculating the minimum variance portfolios, we use the Excel Solver Add-in. Different to the simulation of the weights, the restriction set for the Solver is that the weights of the constituents must be chosen in accordance to their variance and covariance to minimise the overall variance of the portfolio. A further restriction is the non-negativity of index weights since it is not in accordance with the composition rules of the HDAX.

5 Empirical results & analysis

This segment presents the empirical outcomes of the study. We start by presenting the results of concentration ratios, deciles as well as the Hirschman-Herfindahl ratio. Subsequently, the test results of the Garch(1, 1) Model, variance and covariance estimations in addition to the minimum variance portfolio are provided. In the last section we present and discuss the results from the simulation of the different index weightings.

5.1 Concentration measurements

5.1.1 Ratios

In the beginning, we calculated the concentration ratios from C_1 to C_{10} to get a first impression about the concentration of the highest-weighted companies in the index. The ratios are based on the aggregate market weighting of the largest n firms in the HDAX.

The results of Table 1 show the concentration ratios of the ten largest companies calculated each December after the last rebalancing of the year. The index started with an aggregate weighting of around 60% of the ten largest companies. This weighting reached its peak at the end of 2008, one year after the start of the financial crisis. Considering this as the peak, we can see that the concentration of the ten largest companies decreased, apart from a few fluctuations, continuously until an all-time low at the end of 2014 with only 54%.

Year	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
2003	0.1000	0.1700	0.2380	0.3057	0.3724	0.4350	0.4826	0.5267	0.5602	0.5920
2004	0.1000	0.1877	0.2675	0.3359	0.4015	0.4596	0.5154	0.5646	0.6005	0.6353
2005	0.0918	0.1743	0.2424	0.3104	0.3774	0.4425	0.4933	0.5429	0.5865	0.6217
2006	0.0868	0.1716	0.2511	0.3189	0.3754	0.4289	0.4824	0.5266	0.5707	0.6047
2007	0.1000	0.2000	0.2718	0.3436	0.3976	0.4461	0.4929	0.5394	0.5846	0.6204
2008	0.1000	0.1953	0.2638	0.3314	0.3964	0.4534	0.5054	0.5563	0.6031	0.6497
2009	0.0929	0.1803	0.2469	0.3120	0.3748	0.4288	0.4799	0.5290	0.5765	0.6217
2010	0.1000	0.1735	0.2398	0.3019	0.3569	0.4107	0.4619	0.5103	0.5502	0.5801
2011	0.1000	0.1784	0.2422	0.3037	0.3583	0.4117	0.4577	0.5035	0.5477	0.5818
2012	0.0860	0.1695	0.2403	0.3093	0.3707	0.4226	0.4638	0.4996	0.5338	0.5670
2013	0.0821	0.1603	0.2305	0.2969	0.3572	0.4160	0.4534	0.4890	0.5229	0.5540
2014	0.0854	0.1603	0.2280	0.2905	0.3524	0.4094	0.4496	0.4808	0.5107	0.5403
Average	0.0938	0.1768	0.2469	0.3133	0.3743	0.4304	0.4782	0.5224	0.5623	0.5974
High	0.1000	0.2000	0.2718	0.3436	0.4015	0.4596	0.5154	0.5646	0.6031	0.6497
Low	0.0821	0.1603	0.2280	0.2905	0.3524	0.4094	0.4496	0.4808	0.5107	0.5403

Table 1: Concentration ratios, yearly accumulated. C_5 = the weight of the 5 largest constituents in the index; for calculations see equation 7.

Furthermore, we can discover that all ratios from C_1 to C_{10} have lost in terms of weighting over the examined period. C_1 to C_4 have reached the highest concentration at the end of 2007, C_5 to C_8 at the end of 2004 and C_9 and C_{10} at the end of 2008. The greatest rise in concentration of C_{10} took place from 2003 to 2004 when the ratio rose from 59.20% to 63.53% (a 7.32% year over year increase). This year has shown the greatest increase on average over all ratios as well (from 37.83% to 40.68%). Compared to this, the largest decrease in the aggregate concentration of C_{10} has occurred between 2009 and 2010, when it declined from 38.43% to 36.85%. On average, all concentration ratios started decreasing after its peak in 2008, with the largest drop of 5.22% between 2008 and 2009.

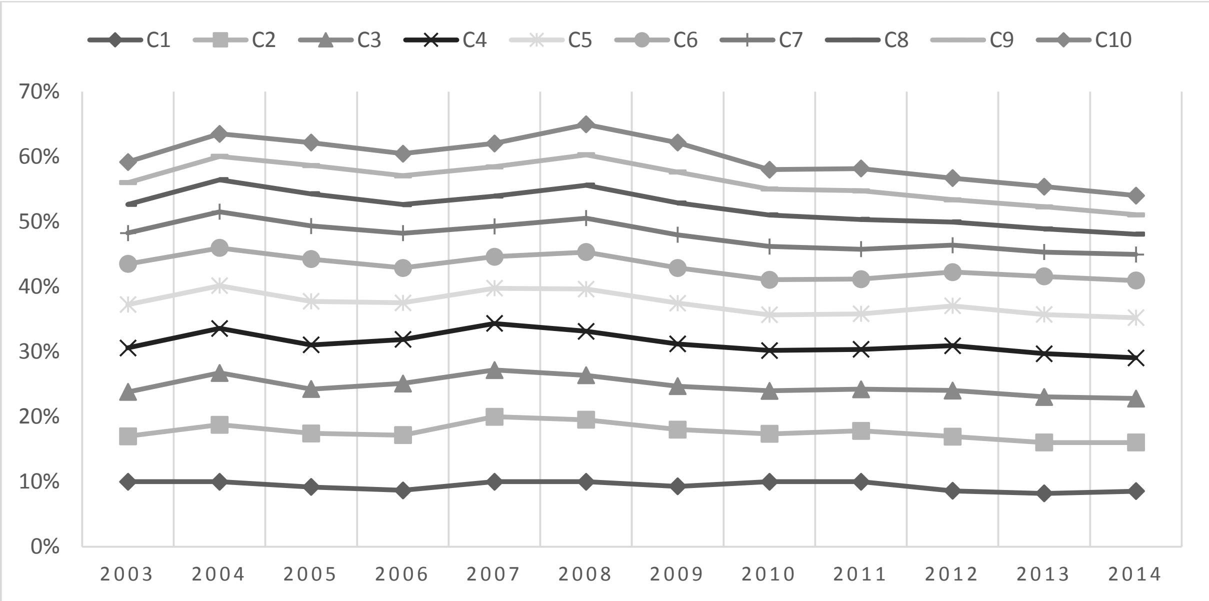

Figure 3: Concentration ratios, yearly accumulated

Overall, we observed a downward trend starting in 2008 of the aggregate weighting of the largest ten companies reducing their weight down to around 54%. This might be due to the financial crisis, which forced the companies to shrink their balance sheet because of a decreasing demand. A further conclusion could be that the constituents 11 to 110 grew proportionally faster in value than the largest ten companies in the index. Nevertheless, the HDAX is still a highly concentrated index with an aggregate weighting of more than 50% in each year since its introduction in 2003.

5.1.2 Hirschman-Herfindahl Index

Another way of measuring concentration is the Hirschman-Herfindahl index (or H ratio), which focuses on the dispersion of firm size for a certain industrial group and is calculated according to equation (8).

The H ratios, which are shown in Table 2, reflect the same pattern as the concentration ratios. The ratio has their highest value in 2008, when the concentration of the ten largest companies reached its peak and is continuously decreasing until the end of 2014. Between 2003 and 2008 the H ratio did not rise gradually, rather it moved in an upward range reaching its peak in 2008.

At the end of 2014, the H ratio confirms with a value of 0.0388 the lowest point of concentration within the index. Nevertheless, compared to the results in the paper of Chelley-Steeley (2008) on the FTSE100, the H ratio of the HDAX shows a higher concentrated HDAX index than its counterpart in the UK.

5.1.3 Deciles

Further, we calculated the aggregate weighting using deciles, which were formed at the end of each year, based on the weighting of every single company. The results are shown in Table 2 and Figure 4. Remarkably the deciles of the six smallest groups are almost flat for the observed period, which

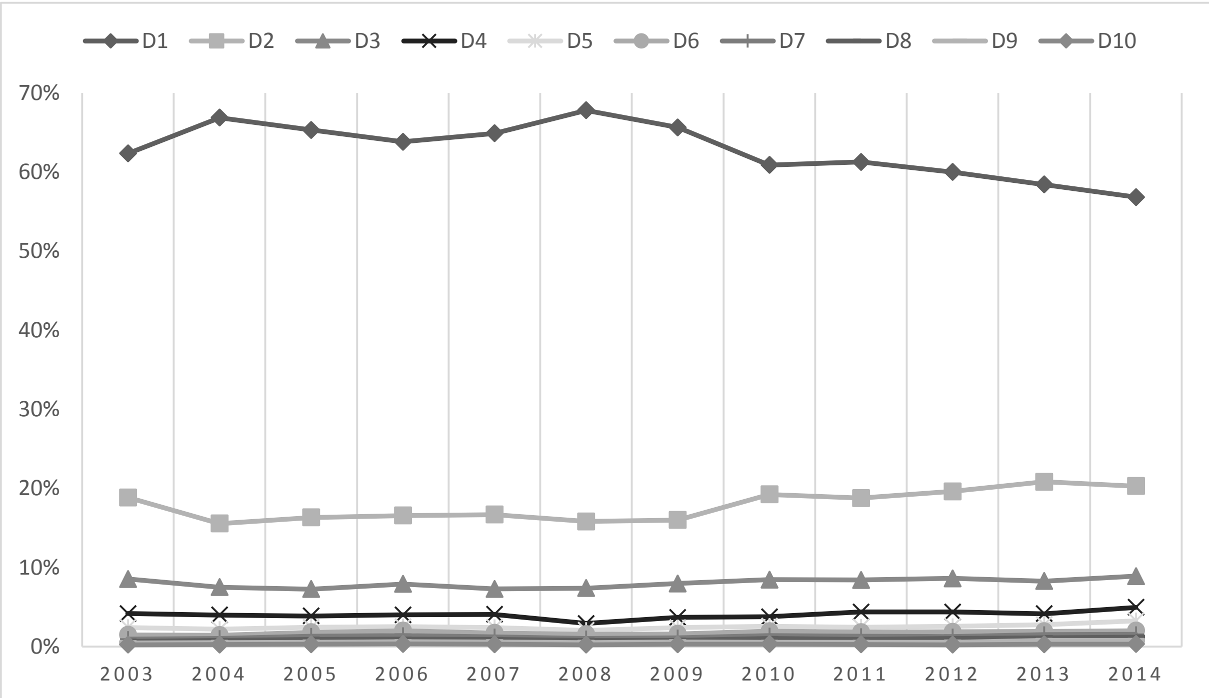

Figure 4: Decile weightings

indicates that they have changed very slightly during the last 12 years. As already shown in the concentration ratios, D1 has constant high weightings with its lowest values in 2013 and 2014. Otherwise the concentration level is always over 60% meaning that up to two third of the whole index weighting is hold by only 11 companies. Furthermore, D1 to D3 (33 companies) amount for up to 90% of the whole index. On the other side, the three smallest deciles D8-D10 never exceed a weighting of more than 2% over time, except for 2013 and 2014. This shows that smaller companies have close to no influence on the performance of the index. Additionally, their impact on the variance is due to their weightings very slim. When looking at the deciles D4 to D7, it is noticeable that these deciles had their smallest values in 2008, when the concentration of the largest deciles was the highest. They consequently started to increase each year until reaching an all-time high by the end of 2014. Finally, it is noticeable that the fluctuation of D1 and D2 is high compared to the other deciles, which are characterised by a high stability over time.

5.1.4 Lorenz curve & Gini coefficient

Our final measurement of concentration, the Lorenz curve as well as the associated Gini coefficient support our findings where higher values imply less equality and therefore a less concentrated index. They are illustrated in Figure 5.

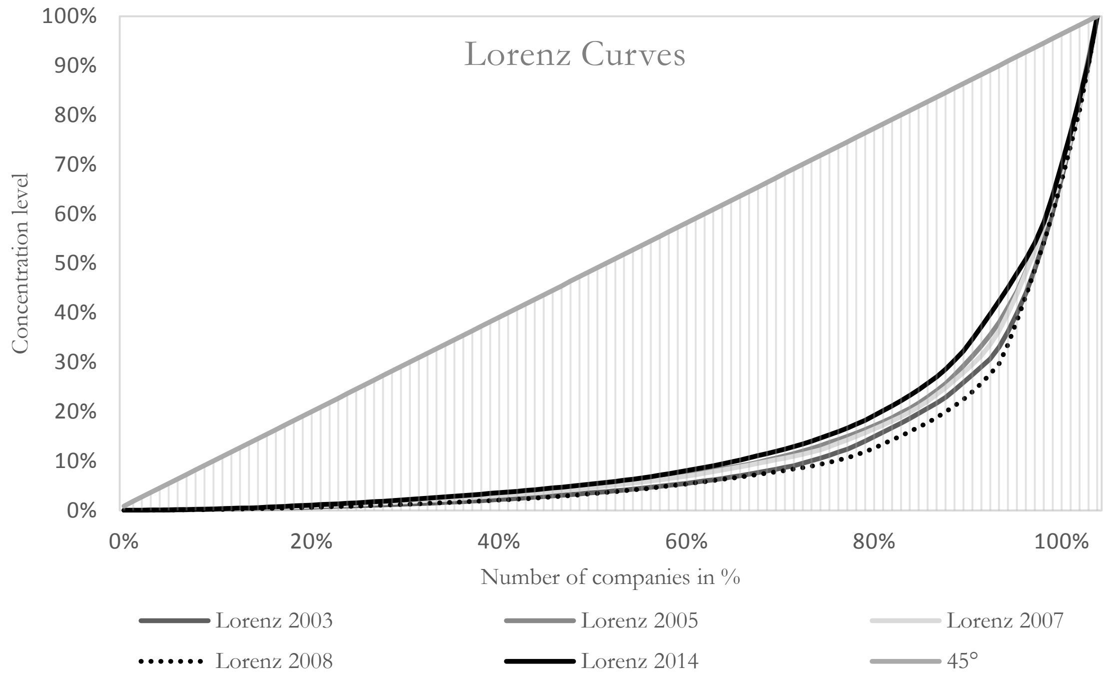

Figure 5: Lorenz curves (for clarity reason not all years are illustrated)

The results of the Gini coefficient as well as the Lorenz curve show once more that the HDAX is a highly concentrated index. The Lorenz curve of 2008, which is the furthest away from the 45° line, shows that companies are more unequal in size than in any other year. Compared to this, the Lorenz curve of 2014 shows the most equality over the whole period, since its line is the closest to the 45° line. Furthermore, the Gini coefficient has its highest value at the end of 2008 and its lowest in 2014, which supports Figure 5 as well. Before 2008, the Gini coefficient rose and fell, while displaying no clear pattern. After reaching its peak in 2008 it decreased constantly until an all-time low in 2014.

Table 2

Weightings of each HDAX decile

Year	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	H	G
2003	0.6234	0.1885	0.0852	0.0416	0.0240	0.0147	0.0092	0.0066	0.0047	0.0021	0.0443	0.7378
2004	0.6688	0.1554	0.0750	0.0397	0.0217	0.0144	0.0102	0.0076	0.0049	0.0023	0.0487	0.7485
2005	0.6531	0.1632	0.0726	0.0387	0.0242	0.0180	0.0130	0.0090	0.0054	0.0028	0.0461	0.7358
2006	0.6383	0.1655	0.0789	0.0399	0.0249	0.0201	0.0138	0.0093	0.0061	0.0032	0.0443	0.7263
2007	0.6490	0.1668	0.0728	0.0405	0.0241	0.0171	0.0124	0.0088	0.0059	0.0027	0.0477	0.7354
2008	0.6780	0.1581	0.0738	0.0291	0.0204	0.0155	0.0100	0.0079	0.0051	0.0021	0.0495	0.7546
2009	0.6564	0.1598	0.0796	0.0367	0.0238	0.0159	0.0111	0.0079	0.0059	0.0029	0.0457	0.7403
2010	0.6088	0.1922	0.0845	0.0375	0.0257	0.0194	0.0137	0.0101	0.0051	0.0029	0.0426	0.7212
2011	0.6127	0.1876	0.0842	0.0436	0.0243	0.0182	0.0128	0.0088	0.0052	0.0026	0.0426	0.7244
2012	0.6001	0.1961	0.0861	0.0437	0.0255	0.0185	0.0125	0.0100	0.0054	0.0022	0.0418	0.7196
2013	0.5842	0.2082	0.0827	0.0413	0.0276	0.0193	0.0152	0.0111	0.0076	0.0028	0.0403	0.7082
2014	0.5681	0.2029	0.0889	0.0496	0.0325	0.0200	0.0154	0.0119	0.0076	0.0031	0.0388	0.6953
Average	0.6284	0.1787	0.0804	0.0402	0.0249	0.0176	0.0125	0.0091	0.0057	0.0026	0.0444	0.7289
High	0.6780	0.2082	0.0889	0.0496	0.0325	0.0201	0.0154	0.0119	0.0076	0.0032	0.0495	0.7546
Low	0.5681	0.1554	0.0726	0.0291	0.0204	0.0144	0.0092	0.0066	0.0047	0.0021	0.0388	0.6953

Table 2: Weightings of each HDAX decile

5.2 GARCH(1, 1) Model

As a measure of the link between index volatility and concentration, we estimated a GARCH(1, 1) Model. The results of the estimation of this model, using daily, weekly and monthly index returns are shown in Table 3. For all index returns δ is either zero or slightly positive, but only significant for daily and weekly returns. This implies that during 2007 and 2008 the return volatility was constant or even slightly higher when concentration was at its peak. Since the results of δ are close to zero, no clear trend can be identified that the level of concentration has a significant influence on the volatility of the index.

	α_0	β_0	δ_0	β_1	β_2
Daily	0.0009***	0.0000***	0.0000***	0.0888***	0.8946***
	(4.8510)	(5.6963)	(3.8802)	(11.1373)	(98.0874)
Weekly	0.0035***	0.0001***	0.0001**	0.1585***	0.7727***
	(3.288)	(4.6801)	(2.2909)	(6.990)	(27.1388)
Monthly	0.0141***	0.0004	0.0014	0.2803*	0.5161***
	(3.3081)	(1.8338)	(1.8439)	(3.1627)	(3.8261)

***	Significance at the 1% Level
**	Significance at the 5% Level
*	Significance at the 10% Level

Table 3: GARCH volatility estimates

5.3 Variances

Table 4 reports the results of the variances of each security in the HDAX index. To get an overview and make the amount of data more manageable, we report the average variances of the 10 deciles based on the market weighting in Table 4. The variance was computed on a five-year rolling basis. The table reports a significant relationship between concentration and volatility. It is noticeable that the five deciles with the highest weighted companies (D1-5) have a much lower average variance than the following five deciles (D6-10) with lower weightings. The same results can be found in the study of Chelley-Steeley (2008). With almost 3%, D10 has the highest average variance from all deciles. This characteristic is also shown in the variance rankings. The total variance maxima over all deciles and all years, except for 2013 and 2014, are either in decile nine or ten, which correspond to the companies with the smallest weighting in the HDAX. When looking at the smallest variance for each of the twelve years, we can observe that in nine out of twelve years the lowest average variance can be found in D1 and D2, including the years 2007 and 2008 that had the highest concentration of the index over the whole observed time period.

The highest average variance with 5.43% is found in 2004 in decile ten. However, the average variance for the deciles 1-10 decreased apart from a few fluctuations continuously from 2003 onwards ending at an all-time low of 0.75% in 2014. An interesting finding was that the variance of the first decile is always lower than the average of the deciles 2-10. This would speak against the hypothesis that a high concentration has an increasing effect on the variance of the index.

Generally, we observe that a high concentration does not imply a high volatility. We rather view that the volatility, respectively the variance is decreasing with increasing concentration. This can be seen in Table 4 since the highest average variances over time can be found in 2003 and 2004, and the lowest in 2013 as well as 2014. The period of 2007 and 2008, which is characterised by the highest concentration, reports an average variance located in between that of the highest period in 2003 and that of the all-time low in 2014.

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Table 4

Monthly volatility of HDAX constituents by decile

	σ(Hdax)	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	Average
1998-2003	0.0593	0.0149	0.0165	0.0095	0.0148	0.0168	0.0322	0.0232	0.0371	0.0346	0.0450	0.0245
1999-2004	0.0560	0.0126	0.0124	0.0115	0.0120	0.0161	0.0271	0.0149	0.0271	0.0333	0.0543	0.0221
2000-2005	0.0537	0.0106	0.0100	0.0110	0.0111	0.0119	0.0135	0.0234	0.0262	0.0153	0.0360	0.0169
2001-2006	0.0466	0.0083	0.0079	0.0079	0.0070	0.0164	0.0121	0.0162	0.0206	0.0185	0.0305	0.0145
2002-2007	0.0356	0.0049	0.0058	0.0070	0.0111	0.0069	0.0163	0.0097	0.0124	0.0254	0.0170	0.0117
2003-2008	0.0446	0.0062	0.0082	0.0102	0.0158	0.0118	0.0106	0.0204	0.0302	0.0234	0.0264	0.0163
2004-2009	0.0516	0.0078	0.0113	0.0164	0.0169	0.0192	0.0155	0.0231	0.0226	0.0249	0.0252	0.0183
2005-2010	0.0543	0.0099	0.0096	0.0191	0.0142	0.0153	0.0159	0.0159	0.0254	0.0220	0.0283	0.0176
2006-2011	0.0564	0.0113	0.0090	0.0207	0.0113	0.0129	0.0210	0.0161	0.0223	0.0196	0.0285	0.0173
2007-2012	0.0688	0.0115	0.0104	0.0186	0.0124	0.0227	0.0149	0.0177	0.0173	0.0150	0.0219	0.0162
2008-2013	0.0452	0.0077	0.0075	0.0157	0.0103	0.0079	0.0106	0.0108	0.0129	0.0104	0.0138	0.0108
2009-2014	0.0390	0.0042	0.0046	0.0075	0.0059	0.0061	0.0078	0.0078	0.0086	0.0095	0.0128	0.0075
Average	0.0509	0.0092	0.0094	0.0130	0.0119	0.0137	0.0165	0.0166	0.0219	0.0210	0.0283	0.0161

Table 4: Monthly volatility of HDAX constituents by decile

5.4 Covariances

A detailed observation of the covariances between the companies included in the HDAX is more significant than the variances, because the number of observable covariances is considerably higher than the number of variances, namely 11,990 covariances in comparison to 110 variances. To make this amount of data more manageable, we computed the average covariance between company i and each of the other 109 stocks over the last five-year period. We then sorted the companies depending on their market weighting in the index and distributed them into 10 deciles. In Table 5, we show the average of the 1,190 covariances for each decile.

The covariances show an even clearer result. The average covariances of component securities peak at the end of 2009, in a period when the concentration had already passed its maximum, but when it was still noticeably high. Nevertheless, we cannot observe any pattern, since the covariance is fluctuating in a range of 0.40% to 0.78% between the periods 2003 to 2012, except for 2007 when the concentration reached its peak. During this year, the average covariance has the second lowest value over the whole observed period with 0.26%; merely at the end of 2014, we report a lower covariance of 0.23%.

In almost all the periods the deciles containing the smaller firms of the HDAX, i.e. deciles five to ten have a higher covariance than deciles one to four. If we compare the average covariance contained in decile one with the average of the other nine deciles we find that over all periods the covariance was significantly lower in decile one than the average of deciles two to ten. Especially between decile one and ten a large difference can be identified.

Overall, we found that a high concentration does not automatically coincide with a high level of covariance; rather we found that there is no visible trend between covariance and concentration but a positive relation between volatility and covariance. Decile ten does not only have the highest average variance but also the highest average covariance.

Table 5

Monthly covariances of HDAX constituents by decile

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	Avg.	D2-10 (Avg.)
1998-2003	0.0064	0.0065	0.0038	0.0054	0.0052	0.0086	0.0066	0.0066	0.0083	0.0086	0.0066	0.0066
1999-2004	0.0055	0.0055	0.0044	0.0042	0.0050	0.0063	0.0045	0.0060	0.0087	0.0094	0.0060	0.0060
2000-2005	0.0049	0.0045	0.0046	0.0032	0.0037	0.0048	0.0053	0.0061	0.0049	0.0077	0.0050	0.0050
2001-2006	0.0039	0.0039	0.0030	0.0029	0.0041	0.0039	0.0036	0.0047	0.0041	0.0057	0.0040	0.0040
2002-2007	0.0020	0.0023	0.0021	0.0027	0.0024	0.0033	0.0021	0.0026	0.0028	0.0032	0.0026	0.0026
2003-2008	0.0045	0.0052	0.0059	0.0073	0.0065	0.0051	0.0087	0.0099	0.0086	0.0083	0.0070	0.0073
2004-2009	0.0053	0.0061	0.0073	0.0084	0.0086	0.0073	0.0086	0.0095	0.0087	0.0085	0.0078	0.0081
2005-2010	0.0056	0.0056	0.0074	0.0069	0.0061	0.0072	0.0061	0.0092	0.0076	0.0083	0.0070	0.0072
2006-2011	0.0061	0.0050	0.0078	0.0057	0.0060	0.0073	0.0067	0.0077	0.0075	0.0081	0.0068	0.0069
2007-2012	0.0061	0.0051	0.0066	0.0058	0.0072	0.0059	0.0070	0.0067	0.0057	0.0071	0.0063	0.0064
2008-2013	0.0033	0.0029	0.0043	0.0032	0.0023	0.0032	0.0034	0.0036	0.0025	0.0030	0.0032	0.0032
2009-2014	0.0022	0.0020	0.0027	0.0019	0.0022	0.0023	0.0026	0.0021	0.0024	0.0025	0.0023	0.0023
Average	0.0047	0.0046	0.0050	0.0048	0.0049	0.0054	0.0054	0.0062	0.0060	0.0067	0.0054	0.0055

Table 5: Monthly covariances of HDAX constituents by decile

5.5 Minimum Variance Portfolios

As a further measurement we investigated the volatility and concentration characteristics of the HDAX by building a minimum variance portfolio. We calculated the minimum variance portfolio weights of the HDAX index for each five-year period between 2003 to 2014. As a result, concentration ratios of these minimum weights are computed.

Table 6 shows the aggregate weightings of each decile based on their minimum variance structure. If the minimum variance portfolio would replicate the composition of the HDAX, the results would be the same as in Table 2. However, the weighting structure of the minimum variance portfolio differs strongly. The weighting of the decile comprising the largest companies vary between zero percent at the end of 2014 up to more than 27% in 2009. Compared to this, D9 and D10 stay almost stable over time and are less volatile. However, the largest change in weightings can be observed for the deciles D2 to D7. On average D1 has lost more than 52% compared to the HDAX weightings over the observed period. The second decile D2 decreased on average by 6.80%. The deciles D3 to D10 have increased their weighting on average by 7.42%, while D8 with 11.24% and D4 with 10.09% have the largest impact. Another interesting fact is that many companies are weighted with zero percentage in the minimum variance portfolio, which is due to their volatile return structure.

When comparing the Gini coefficient and the H ratio of the minimum variance portfolios with the original HDAX composition, we observe that the Gini coefficient of the MVP is significantly lower for each year compared to the original one. A different result can be observed for the H ratio, which shows slightly higher results in each year. The Gini coefficient can be explained through a significantly lower concentration dispersion in the deciles of the MVP. Especially decile one lost proportionally the most of its weighting to a more equally weighted index composition. A reason for the increasing H ratio of the MVP compared to the original index composition could be the fact that we did not introduce any restrictions regarding a maximum weight for each single company as it is the case for the HDAX. As a result companies can have a weighting of more than ten percent in the portfolio.

Furthermore, the findings also show that the weighting structure of the HDAX based on its market values does not mirror a minimum variance portfolio. Additionally, we found large changes in concentration, but no trend on the variance. All in all our results support our preceding findings that there is no evidence for a relationship between concentration in the HDAX and its volatility.

Table 6

Minimum variance concentration ratios

Year	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	H	G
2003	0.0718	0.0559	0.1402	0.0822	0.1971	0.1034	0.0340	0.1256	0.1331	0.0568	0.0604	0.2622
2004	0.0113	0.0157	0.0954	0.3870	0.0702	0.1584	0.2492	0.0083	0.0000	0.0046	0.1239	0.6252
2005	0.0062	0.1101	0.0507	0.3049	0.1923	0.0201	0.1074	0.1753	0.0208	0.0123	0.0878	0.3371
2006	0.0711	0.0000	0.2622	0.1894	0.0029	0.0886	0.0588	0.2875	0.0396	0.0000	0.0713	0.3974
2007	0.1176	0.0866	0.1683	0.0914	0.0137	0.0208	0.1281	0.2520	0.0571	0.0643	0.0671	0.1565
2008	0.2218	0.2013	0.1122	0.0403	0.1395	0.2849	0.0000	0.0000	0.0000	0.0000	0.0959	0.3685
2009	0.2741	0.1688	0.1309	0.0379	0.0287	0.1900	0.1495	0.0029	0.0084	0.0089	0.0901	0.2683
2010	0.1722	0.1655	0.0637	0.1215	0.3345	0.0360	0.0579	0.0487	0.0000	0.0001	0.1207	0.3031
2011	0.1518	0.1674	0.1398	0.0994	0.1230	0.1177	0.1314	0.0614	0.0012	0.0068	0.0786	0.0415
2012	0.1209	0.1150	0.2972	0.0634	0.1060	0.1360	0.0489	0.0732	0.0325	0.0069	0.0737	0.1800
2013	0.0121	0.0884	0.0469	0.1003	0.1326	0.0666	0.2822	0.1921	0.0742	0.0046	0.0784	0.2404
2014	0.0000	0.1537	0.1006	0.1758	0.1146	0.0760	0.0565	0.2304	0.0816	0.0108	0.0744	0.1674
Average	0.1026	0.1107	0.1340	0.1411	0.1212	0.1082	0.1086	0.1215	0.0374	0.0147	0.0852	0.2790

Table 6: Minimum variance concentration ratios

5.6 Simulation of weights

Finally, as a last step of our study, we measured the sensitivity of the variance in relation to the concentration level by simulating different levels of concentration and applying these to the HDAX index. Table 7 shows a five-year rolling variance for the three artificial HDAX indices, starting with an all-time high in 2003 with a variance of 5.9% for the low and 6.1% for the high concentration. Interesting to see is that with the beginning of the financial crisis in 2007 and in its following years, high concentration in the HDAX led to a lower variance than a low concentrated index. This might be due to the fact that the volatility of companies with a large market value is less affected by turbulent economic times than those of smaller companies. A noteworthy finding shown in Table 7 is that altering the weighting structure, from low to high concentration, has little to no effect on the variance. The variance calculated with the three different weightings is surprisingly uniform. Column five of Table 7 highlights the difference between the variance of the high and the low concentrated portfolio. It shows that during the years of 2008 to 2010, when the concentration level was at its peak, a lower concentrated index would not have led to a lower variance. Furthermore, during the other years, the differences in the volatility between the synthetic HDAX indices are close to zero.

Weights/Years	Low	Intermediate	High	Low-High	Equal
1998-2003	0.0587	0.0598	0.0606	-0.0020	0.0591
1999-2004	0.0535	0.0554	0.0568	-0.0033	0.0561
2000-2005	0.0517	0.0528	0.0535	-0.0018	0.0512
2001-2006	0.0464	0.0466	0.0471	-0.0007	0.0459
2002-2007	0.0350	0.0352	0.0353	-0.0003	0.0370
2003-2008	0.0451	0.0448	0.0446	0.0005	0.0601
2004-2009	0.0527	0.0514	0.0508	0.0019	0.0636
2005-2010	0.0538	0.0539	0.0535	0.0003	0.0603
2006-2011	0.0559	0.0560	0.0575	-0.0016	0.0593
2007-2012	0.0576	0.0578	0.0590	-0.0013	0.0573
2008-2013	0.0456	0.0457	0.0462	-0.0006	0.0408
2009-2014	0.0390	0.0399	0.0404	-0.0014	0.0349
Average	0.0496	0.0499	0.0504	-0.0008	0.0521

Table 7: Simulated weights

For all three indices the average variance ranges around 5%. As a last measure we assigned each component of the HDAX the same weighting to minimise the level of concentration. The variance of this index can be found in the last column of Table 7. It shows that an equal weighting of all components leads to a higher variance. This leads to the conclusion that low concentration levels can have a negative impact on the variance of the index.

6 Conclusion

In the final section of the study the conclusion of the empirical findings is presented and evaluated. Additionally, possible shortcomings of this study and further research questions are being discussed.

6.1 General

The purpose of this paper was to examine the influence of the HDAX composition between 2003 and 2014 on the index variance. We applied several statistics to measure the concentration of the HDAX over the observed time period.

First, through the creation of concentration ratios and by segmenting the 110 index constituents into deciles we find a highly concentrated index with the top 10 companies making up on average 60% of the index weighting. However, a downward trend can be observed starting in 2008. The Gini coefficient and Lorenz curve, used to investigate for inequality in the composition, confirm the findings that the HDAX is a highly concentrated index with its concentration peak in 2008. The most equality can be observed in 2014. Although concentration levels were high in the years of 2007 and 2008, our Garch (1, 1) Model indicates no significant influence between high concentration and high index volatility.

When analysing the deciles, it is notable that the six smallest groups have close to no influence on the development of the index since they only make up on average 7.5% of the index, meaning that the biggest company has a higher influence than the 66 smallest constituents of the index combined. Instead decile one (the 11 biggest companies) makes up on average around 2/3 of the index weighting.

Examining the variance of the constituent companies we find that a high concentration within the index has no negative effect on the index volatility. In our case, we rather observe that the volatility is decreasing with increasing concentration. This is due to the fact that with higher concentration, large companies with lower volatility have stronger influence on the overall index variance. When studying all 11,990 covariances we cannot identify a clear trend between an increase of concentration and the covariance. However, we can find that the smaller companies have a considerably larger covariance than the larger constituents of the index.

To show how insignificant the level of concentration is on the volatility of the index we created a variance-covariance matrix for the 110 constituents of the HDAX to simulate different weightings. It shows that the concentration has little to no effect on the variance, it is rather remarkably uniform. However, an equal weighting of the index, which minimises the concentration, increases the index variance substantially.

Completing the picture, we created minimum variance portfolios with the constituents of the HDAX and compared concentration level and variance with the actual observed concentration levels. We find that the weighting structure of the HDAX based on market values does not lead to a minimum variance. However, regarding the concentration measures of the minimum variance portfolio we can see large changes in concentration with no effect on the variance. Hence, no trend can be found and the results confirm no relationship between the concentration in the HDAX and the corresponding volatility.

6.2 Discussion

The study of concentration risk in the HDAX has shown that a high level of concentration does not coincide with a high volatility at the same time. However, in this study we investigated the relationship between concentration risk and volatility. Further studies in this field should also take into account the influence of the concentration on the return of an index. This is due to the fact that fund managers are often benchmarked against a certain level of return instead of a level of volatility (Dimson et al., 2009).

A possible shortcoming of this study might lay in the calculation of the minimum variance portfolio, since minimum variance optimisation is very sensitive to errors in the estimates of the inputs (Chopra, 1993). Chopra showed that if the input parameters are changed a little, the composition of the optimal portfolio could change heavily.

One more factor that needs to be critically discussed is the relevance of a five-year backwards looking variance as it was used for our calculations. This caused the problem that for some companies no five-year stock return data was available since they have not been listed long enough on the stock exchange to fulfil these requirements. This led to different time horizons in the calculation of the variance. Another question to ask is how significant is a five-year rolling variance calculation in terms of predicting future development, since there are no empirical results proving the assumption of a five-year backward-looking timeframe.

A last possible shortcoming that we would like to point out is the fact that the index was only observed as a whole. The behaviour and concentration of single industries within the HDAX during different time periods was not studied. Further research could investigate if the concentration of certain industries within an index (e.g. cyclical vs. non-cyclical) is significant in influencing the index volatility.

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Appendices

Appendix A: HDAX return daily

Dependent Variable: HDAX_RETURN_DAILY
Method: ML - ARCH (Marquardt) - Normal distribution
Sample: 2/21/2003 12/31/2014
Included observations: 3094
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1) + C(5)*DUM_DAILY

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000867	0.000179	4.851023	0.0000
		Variance Equation		
C	2.34E-06	4.11E-07	5.696258	0.0000
RESID(-1)^2	0.088838	0.007977	11.13725	0.0000
GARCH(-1)	0.894560	0.009120	98.08740	0.0000
DUM_DAILY	2.14E-06	5.51E-07	3.880190	0.0001

R-squared	-0.000989	Mean dependent var	0.000440
Adjusted R-squared	-0.000989	S.D. dependent var	0.013588
S.E. of regression	0.013595	Akaike info criterion	-6.114324
Sum squared resid	0.571654	Schwarz criterion	-6.104568
Log likelihood	9463.860	Hannan-Quinn criter.	-6.110820
Durbin-Watson stat	2.003131		

Appendix B: HDAX return weekly

Dependent Variable: HDAX_RETURN_WEEKLY
Method: ML - ARCH (Marquardt) - Normal distribution
Sample: 3/26/2003 12/31/2014
Included observations: 615
Convergence achieved after 15 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1) + C(5)
*DUM_WEEKLY

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.003543	0.001078	3.288470	0.0010
Variance Equation				
C	5.15E-05	1.10E-05	4.680076	0.0000
RESID(-1)^2	0.158482	0.022674	6.989520	0.0000
GARCH(-1)	0.772713	0.028473	27.13875	0.0000
DUM_WEEKLY	6.29E-05	2.74E-05	2.290943	0.0220

R-squared	-0.002166	Mean dependent var	0.002199
Adjusted R-squared	-0.002166	S.D. dependent var	0.028911
S.E. of regression	0.028942	Akaike info criterion	-4.426844
Sum squared resid	0.514321	Schwarz criterion	-4.390896
Log likelihood	1366.254	Hannan-Quinn criter.	-4.412865
Durbin-Watson stat	2.128695		

Appendix C: HDAX return monthly

Dependent Variable: HDAX_RETURN_MONTHLY
Method: ML - ARCH (Marquardt) - Normal distribution
Sample: 2003M03 2014M12
Included observations: 142
Convergence achieved after 62 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1) + C(5)
*DUM_MONTHLY

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.014114	0.004267	3.308095	0.0009
		Variance Equation		
C	0.000433	0.000236	1.833812	0.0667
RESID(-1)^2	0.280343	0.088640	3.162731	0.0016
GARCH(-1)	0.516128	0.134897	3.826087	0.0001
DUM_MONTHLY	0.001356	0.000736	1.843886	0.0652

R-squared	-0.007272	Mean dependent var	0.009284
Adjusted R-squared	-0.007272	S.D. dependent var	0.056844
S.E. of regression	0.057050	Akaike info criterion	-3.070591
Sum squared resid	0.458911	Schwarz criterion	-2.966512
Log likelihood	223.0119	Hannan-Quinn criter.	-3.028297
Durbin-Watson stat	1.733106		