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Overview of Disc Brakes and Related Phenomena - a review

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Abstract: Disc brakes have evolved over time to be a reliable method of decelerating and stopping a vehicle. There have been different designs of disc brake systems for different applications. This review gives a detailed description of different geometries of the components and the materials used in a disc brake system. In spite of all the improvements, there are still many operational issues related to disc brakes that need to be understood in a greater detail and resolved. There has been a lot of research going on about these issues and at the same time different methods are being proposed to eliminate or reduce them. There has also been an intensive fundamental research going on about the evolution of tribological interface of disc-pad system. One major purpose of the present paper is to give a comprehensive overview of all such developments.

Keywords: disc brake, disc geometry, pad geometry, disc-pad tribology, brake fade, brake noise.

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1 Introduction

A vehicle requires a brake system to stop or adjust its speed with changing road and traffic conditions. The basic principle used in braking systems is to convert the kinetic energy of a vehicle into some other form of energy. For example, in friction braking it is converted into heat, and in regenerative braking it is converted into electricity or compressed air etc. During a braking operation not all the kinetic energy is converted into the desired form, e.g. in friction braking some energy might be dissipated in the form of vibrations. In [1] a brief description of different brake systems used in a vehicle is given.

Two type of friction brakes, drum brakes and disc brakes, are widely used. Disc brakes as compared to drum brakes cool faster, due to larger swept area and relatively higher exposure to air flow, and show self cleaning ability due to centrifugal forces [2]. Due to these reasons and some other advantages disc brakes have become the universal

Figure 1: A simplified disc brake with the terminology in common use.

choice for front brakes on cars [2] and are also expected to dominate the truck market in the near future [1].

This review paper consists of five major sections: Introduction, Disc brakes, Tribology, Operational issues and Conclusions. In the 'Disc brakes' section, different parts of a disc brake system are presented. Various configurations of parts, and material are also discussed. In the 'Tribology' section, changes happening at the contact interface are described. In the 'Operational issues' section, the issues related to disc brakes encountered during brake application e.g. fade and noise will be described. Finally, in 'Conclusion' section, some concluding remarks are given.

2 Disc brakes

In a disc brake system, a set of pads is pressed against a rotating disc and due to friction, heat is generated at the disc-pad interface. This heat ultimately transfers to the vehicle and environment and the disc cools down. A simplified disc brake is shown in figure 1 with the terminology which is in common use. The pad which is nearer to the center of the vehicle is called the inboard pad while the one that is away is called the outboard pad. Similarly friction surface of the disc which faces towards vehicle is called inboard cheek and the one which faces away is called outboard cheek. The edge of the pad which comes into contact with a point on disc surface first is called leading edge while the edge which touches that point last is called trailing edge. The edge of the pad with smaller radius is called inner edge while the one with larger radius is called outer edge.

A disc brake assembly consists of following major components: brake disc, pad, underlayer, back plate, shim and caliper. Now these components will be described in more detail.

2.1 Brake disc

Brake disc, also called brake rotor, is fixed to the axle, so it rotates with the same speed as the wheel. Braking power of a disc brake is determined by the rate at which kinetic energy is converted into heat due to frictional forces between the pad and the disc. For an efficient brake design, it is also important that heat is dissipated as quickly as possible otherwise the temperature of a disc might rise and affect the performance of a disc brake. So to get an optimum performance in demanding applications, ventilation is introduced in the brake discs which increases the cooling rate. Brake discs could be divided in two categories:

1. Solid brake discs
2. Ventilated brake discs

A solid brake disc is the simplest form and consists of a single solid disc. In a ventilated disc, vanes or pillars or both separate two annular discs and provide a passage for the air to flow. Ventilated brake discs increase the cooling rate and result in lower surface temperature. This lower temperature reduces the risk of brake fade and also helps in reducing wear of the disc and pad. Both of these designs are constructed with or without a mounting bell. A mounting bell increases the distance from the friction surface to axle and the surface area of the disc which improves cooling [3] and therefore it helps to protect the wheel bearings from the high temperature generated due to braking operation. A schematic description of these two types of discs is given in figure 2.

When mounting bell is not a part of the brake disc then this multi-part disc configuration is called hybrid or composite brake disc. In this design, the disc is sometimes called the braking or friction ring. There are different ways to join a mounting bell and a friction ring mainly depending upon the material of the disc. In figure 3, two methods (patented [4, 5]) used for joining a friction ring and a mounting bell by using a connecting element are shown. In figure 3a the connecting element is a special threaded bolt which is screwed to the mounting bell and free to slide in radial direction inside the friction ring. This bolt is usually made of steel which could experience corrosion and heat could conduct quickly to the mounting bell. In figure 3b the connecting element is made of a ceramic material to avoid the corrosion problems and reduce the heat transfer to the bell. The head of the ceramic pin is cast into the mounting bell. In another design (patented [6]), shown in figure 4, several projecting teeth on the inner periphery of friction ring are finely machined and then mounting bell is casted so that these teeth are embedded into the bell material. In this design mounting bell is usually made of a light alloy e.g. aluminum or magnesium. One major advantage of the hybrid brake disc is the relative freedom of expansion of the friction ring which results in lower thermal distortion [7].

Different configurations of vanes and pillars are used in ventilated brake discs. Each configuration gives a unique airflow pattern. Some of the configurations used on the ventilated discs are the following: straight radial vanes, curved vanes, diamond and teardrop pillars (DTDP), and arcuate vanes. Figure 5 shows three different configurations used for ventilation. In all of these, cooling air enters at the inner periphery and leaves the disc at outer periphery. One disadvantage with these configurations is that high stresses develop near the inner periphery primarily due to the inlets [8, 9]. This could be a problem when a disc is used in a demanding situation.

Figure 2: Schematic representation of different brake discs.

Figure 3: Simplified representation of joining of friction ring and mounting bell with connecting elements.

Figure 4: Simplified representation of integrally casting a mounting bell with a friction ring [6].

Straight radial vanes

Straight radial vanes are the very basic design for ventilated discs. They do not have good cooling efficiency but they can be used either on right or left side of a vehicle without any concern about direction of rotation.

Curved vanes

A disc with curved vanes has better pumping action as compared to a disc with straight radial vanes [10]. A disadvantage of having curved vanes on a disc is that discs used on the right and left side of a vehicle must be distinguished as their function depends on the direction of rotation.

DTDP

In a DTDP configuration, pillars resembling the shape of a diamond and a teardrop are arranged. The shape, size and relative locations of the pillars can be configured for maximum heat transfer rate [11].

Arcuate vanes

In this design (patented [12]), cooling air both enters and leaves the disc at outer periphery as shown in figure 6. This configuration overcomes the problem of disc weakness at inner periphery. Here vanes and pillars are arranged so that the disc can be used in both directions. The splines at the inner periphery are used to mount the disc to wheel hub by engaging corresponding splines.

In some designs, a disc is cross-drilled or cross-slotted to increase the cooling rate of the disc but this causes the concentration of stresses near drilled holes and slots [13].

Figure 5: Schematic representation of different configurations used for ventilated discs, showing only 30° segments and one annular ring removed.

Figure 6: Arcuate vane design, arrows show air flow direction for counter clockwise rotation of disc [12].

Figure 7: An assembly of pad, back plate and shim.

Figure 8: Schematic representation of different configurations for a brake pad.

2.2 Brake pad

A brake pad consists of a friction material which is attached to a stiff back plate. Figure 7 shows a brake pad attached to a back plate. Sometimes the friction material and back plate together are called a brake pad. A brake pad usually incorporates slots on its face and chamfers at the ends. Figure 8 shows different configurations of pads. A pad can have more than one slot and it could be arranged in different orientations. One purpose to incorporate chamfers and slots is to reduce squeal noise [14, 15, 16]. Relatively higher temperature at the pad surface than the interior will result in convex bending of the pad [17, 18]. A slot will allow the material to bend and help avoid cracks. Furthermore it facilitates to clean the dust collected between disc and pad surfaces by offering an escape.

Figure 9: Schematic representation of different parts that make up a pad assembly.

2.3 Underlayer

Sometimes an additional layer of material, called underlayer or substrate, is placed between friction material and back plate as shown in figure 9. Its main purpose is to damp vibrations originating at the disc-pad interface [19, 20, 21].

2.4 Back plate

A back plate is used to support the friction material and transmit the actuation force. The friction material is mostly attached to the back plate in two ways, adhesive bonding and mechanical retention. Mechanical retention can be achieved in different ways. One way is to weld studs to the back plate that protrude into the friction material as shown in figure 10a. Another way (patented [22]) is to gouge hooks on the surface of back plate as shown in figure 10b. In a similar way (patented [23]) undercuts are created in the back plate which mechanically engage the friction material. Adhesive bonding can delaminate during service so mechanical retention systems are preferred.

2.5 Shims

Shims are laminates of metal and viscoelastic materials. They are placed between a back plate and a piston or calliper housing (in the case of a floating caliper). Their purpose is to dampen the vibrations in the disc-pad system [24]. They are usually attached to the back plate with an adhesive or assembled mechanically. Figure 7 shows a shim attached to a back plate mechanically. Shims can be constructed in various ways and materials e.g. steel core with viscoelastic material on both sides or viscoelastic material core with steel on both sides as shown schematically in figure 11.

Figure 10: Schematic representation of different mechanical retentions systems used on back plates.

Figure 11: Schematic representation of typical shim configurations.

Figure 12: Schematic representation of a fixed caliper.

2.6 Brake caliper

A brake caliper is an assembly which houses the brake pads. In addition it also houses the pistons and provides the channels for the brake fluid which actuates the pistons. There are two types of calipers, fixed and floating. A fixed caliper does not move relative to the brake disc and houses the pistons on both sides of the disc as shown in figure 12. When pressure is exerted both pistons move and push the brake pads. A floating caliper houses the piston only on one side of the disc as shown in figure 13. When pressure is exerted, the piston moves and pushes the inner brake pad. When the pad contacts the disc surface, caliper moves in the opposite direction so that outer pad also contacts the disc surface. In this design, inboard pad is also called piston-side pad and outboard one is called finger-side pad.

In general, the inner and outer pads show different contact pressure distributions and wear behaviors [25, 19] due to different support and actuation systems and difference in thermal deformations of inboard and outboard cheeks of a disc [8].

2.7 Materials

A variety of materials is used for the different components of the disc brake system. The choice of materials mainly depends upon the application and characteristics desired.

2.7.1 Disc materials

Grey cast iron with a predominantly pearlitic matrix is the widely used material for brake discs [26]. The benefits of using it as a disc material are good castability and machinability, high thermal conductivity and heat capacity, resistance to brake fade and lower cost [27].

There is an interest in the industry to use lighter materials for the disc so that it contributes less to the overall weight of the vehicle and ultimately improves the fuel

Figure 13: Schematic representation of a floating caliper.

consumption. Another reason for lighter disc is that the brake discs are part of the unsprung mass of the vehicle, so its reduction also adds to driving comfort.

One way to reduce the weight is to use aluminium mounting bell and cast iron braking ring in a hybrid brake disc. These two parts can be connected with either integrally casted grey iron braking ring with aluminium bell [28] or by mechanical connections e.g. integrating radial steel inlays [7]. Friction ring is made of cast iron to take advantage of its superior friction and thermal properties while mounting bell is made of aluminium to reduce the overall weight.

To reduce the weight further, lighter materials with suitable properties are required to be used as a braking ring. As the heat is generated at the surface of a disc due to frictional forces, it is desired that for a given heat input, material temperature rise is the minimum i.e. it should have maximum possible volumetric heat capacity (i.e. density $\times$ specific heat capacity). During a short braking this is very important as a significant amount of heat is stored [29]. However during long brakings, it also becomes important that this heat conducts to the core of the disc quickly from the friction surface i.e. disc material should have high thermal conductivity. Furthermore, during long and repeated brakings it is also important that the heat conducts quickly relative to stored heat i.e. it should have high thermal diffusivity (i.e. thermal conductivity/volumetric heat capacity). Other properties desired are friction stability, resistance to corrosion, low wear and lower coefficient of thermal expansion [30].

Aluminum based metal matrix composite materials are an alternative for the braking ring. They offer good wear and corrosion resistance and considerable weight saving as compared to cast iron [31]. They also have higher thermal conductivity and diffusivity [32]. One drawback is the higher coefficient of thermal expansion as compared to cast iron [32]. Another major drawback is limited temperature resistance and for this reason they have not gained wide acceptance [33, 34].

Ceramic Matrix Composites (CMCs) based on carbon fibres and matrices of silicon carbide are another choice for brake discs due to their superior tribological properties in comparison to grey cast iron [35]. Their key characteristics are lower density and

coefficient of thermal expansion as compared to cast iron, high and widely constant coefficients of friction, and high fading stability [34]. One drawback with CMCs is their lower volumetric heat capacity which is compensated by slightly larger discs [34]. They are being used in high performance automobiles but their prices are currently very high as compared to cast iron discs [36]. In [34] a method of joining a CMC disc to a mounting bell is shown.

2.7.2 *Friction materials*

From functional and safety point of view following demands are put on the behavior of friction materials [17, 37, 38]

- High coefficient of friction
- Stable coefficient of friction irrespective of temperature, velocity, pressure, humidity, wear, corrosion and water spraying
- Low wear rate of the friction material and long life
- Low wear rate of the disc
- Smooth braking without noise and vibrations
- Regeneration of original properties after severe braking and with aging
- Environment friendly raw materials
- Low cost

This puts tremendous demands on the material selection process. To fulfill so many demands friction materials are made from many constituents and hence are termed composites. These are considered to be anisotropic [18, 19]. There could be up to 25 different components [3]. The constituents are typically classified into following four categories [39]:

Binders

A binder holds different components of a friction material together. Thermosetting phenolic resins are usually used as binders, with the addition of rubber for increased damping properties. The binder resin strongly affects fade and wear resistance of a friction material [40].

Reinforcing fibres

Reinforcing fibres provide mechanical strength. Asbestos fibres were widely used as reinforcing material but has been replaced with other materials due to environmental problems connected to asbestos [41]. Other materials used as reinforcing fibres include glass, aramid, ceramic and various metals e.g. copper, steel, iron, brass, bronze and aluminum [42]. Fibres are preferentially mixed in tangential or radial direction of the pad and not in the axial direction [43]. This is to keep the heat away from the back plate as fibres, especially metallic, have usually high thermal conductivity. If excessive heat flows in axial direction it could eventually reach the hydraulic brake fluid and may cause loss in brake effectiveness.

Table 1 Classification of friction materials, adapted from [19]

	Metal content [wt%]	Metal type
Semi-metallic	≥ 40	ferrous
Low-steel low-metallic	≤ 15	ferrous and non-ferrous
No-steel low-metallic	≤ 15	non-ferrous
European metallic	15-40	ferrous and non-ferrous

Fillers

Fillers improve the manufacturability of a friction material. Some examples are cashew dust, mica, calcium carbonate and barium sulphate.

Frictional additives

Friction additives control the frictional coefficient and wear rate. They are divided in two categories: lubricants and abrasives. Lubricants stabilize the friction coefficient especially at high temperatures. Abrasives increase friction coefficients however they also cause a rise in wear rate of a disc. They remove iron oxides as well as other undesirable surface films from the disc face [39]. Graphite and metal sulphides are typically used as lubricants while metal oxides and silicates are used as abrasives.

Friction materials have different proportions of these basic constituents. The exact composition of commercially available friction materials is mostly a trade secret. Blau [44] presents a survey of commercial brake materials.

Friction material are classified differently in different works. In [2], they are classified as: organic, semi-metallic, metallic, synthetic and ceramic. In [19], they are classified into semi-metallic, low-steel low-metallic, no-steel low-metallic and European metallic depending on ferrous and non-ferrous metal content as elaborated in table 1. No-steel low-metallic friction material is also known as non-asbestos organic (NAO).

2.7.3 Back plate materials

For the back plate usually steel is used but aluminium is also used for its lower weight.

2.8 Energy dissipation

Consider a vehicle with a total weight of 2000 [kg] moving at a speed of 100 [km/h] which is fitted with four disc brake systems. Assume that it is braked and stopped in 5 [s]. By ignoring smaller contributions, e.g. rotational kinetic energy of different parts, kinetic energy of the vehicle, expressed as $\frac{1}{2}mv^2$, is 771.6 [kJ]. So by assuming that all of this energy is converted to heat, average dissipation rate is 154.3 [kW]. As the brakes mounted on front axle approximately have 60-80% of a vehicles braking power [2], so each front brake might be dissipating about 60 [kW] on average. But this power is not uniformly distributed over the pad surface as real contact area is approximately 20% of the apparent contact area [45]. This could be compared to the engine power of about 110-140 [kW] available in a vehicle with a 2.4 liter gasoline engine.

3 Tribology

Brake performance depends not only on the thermal and mechanical properties of disc and friction materials but it is also affected by the topography of the mating surfaces and the third body formed as a result of wear processes [45, 38]. So it becomes important to have a good understanding of the changes happening at the contact interface to fully understand the behavior of brakes. Before digging deeper into the tribology of disc brakes some fundamentals of friction and wear will be described.

3.1 Fundamentals of friction and wear

A tribological system is characterized by the frictional force and wear of the contacting surfaces. In many engineering applications, friction is an unwanted phenomenon but in the case of brakes it is very much desired. As the basic working principle of a brake is the resistance force due to dry friction between rotor and pad so it deserves special attention.

3.1.1 Friction

Friction can be described as resistance to the relative sliding motion of bodies in contact. An apparently polished and flat surface of a solid is not perfectly flat and contains irregularities if observed at magnification. These irregularities usually are found in the form of protrusions and depressions, called asperities and valleys, respectively. When one solid is pressed against another one, contact occurs at discrete contact spots. The sum of the areas of all contact spots constitute the real area of contact and this is usually only a small fraction of the apparent area of contact [46].

Friction is a complex phenomenon and result of different mechanisms at the contacting surfaces at different scales. Some of the main mechanisms of dry friction are: adhesion, elastic and plastic deformation of asperities, ratcheting, fracture of asperities and third body mechanism [47].

As friction has strong influence on the performance of a brake system, its accurate representation in the form of a mathematical model is very important for simulating the thermomechanical behavior. The goal of a friction model is to have a good agreement between predicted behavior and experimental observations. In general, many models of friction have been proposed in literature but non of them has succeeded to achieve wide acceptance [48]. So in many applications centuries old classic laws of friction are used extensively.

Classic laws of friction

In figure 14, a body is shown sliding on a flat surface of a fixed rigid support in the direction t with a velocity v_t . Here F is the resultant normal force acting from the sliding body to the support and F_r is the friction force. Classic laws of friction are described as [49]:

1. The friction force is proportional to the normal contact force,

$$\|F_r\| = \mu F, \quad (1)$$

Figure 14: Schematic of a body sliding on a flat surface of a rigid support.

where μ is the coefficient of friction (CoF) and before the start of sliding motion it is called static coefficient of friction (μ_s) and during motion it is called kinetic coefficient of friction (μ_k).

2. The coefficient of friction is independent of the apparent area of contact.
3. μ_s is greater than μ_k .
4. μ_k is independent of the sliding velocity.
5. When sliding occurs the friction force opposes the relative motion, expressed as

$$\mathbf{F}_r = -\mu F \frac{\mathbf{v}_t}{\|\mathbf{v}_t\|}. \quad (2)$$

First two laws are called Amontons' laws of friction and fourth law is called the Coulomb's law of friction [50]. Deviations from some of these laws have been reported for some applications [49].

The real area of contact is almost linearly proportional to the load [47]. In general it can be said that friction increases with an increase of real area of contact.

3.1.2 Wear

Dry friction causes the wear of the surfaces of the friction partners. This results in the formation of a third body and also changes the topography of the surfaces of friction partners. So for a given system wear plays a significant role on the evolution of CoF. Typical wear mechanisms are: adhesion, abrasion, fatigue, corrosion, chemical wear etc. [43, 51]. To compare different friction materials, specific wear rate is often used in literature, which is the wear amount normalized by the friction energy [52].

In the past, many wear models have been proposed but most models are system specific i.e. for a particular material of friction partners, contact geometry, operating and environmental conditions [53]. Meng and Ludema [51] performed an extensive review of wear models and analyzed their applicability. Currently there is no model available that can predict wear given the tribological conditions and material properties [51, 53]. In practice, an equation is usually used to correlate data from wear experiments and then this equation with its parameters can be used as a wear model. One of the most popular model for wear is the Archard's wear law.

Archard's wear law

According to Archard's wear law [54, 55] wear rate is proportional to the normal force and sliding distance. This law is expressed as

$$\Delta W = kFs, \quad (3)$$

where ΔW is the worn volume, k the coefficient of wear, F the applied load and s is the sliding distance.

3.2 Tribological interface

Wear plays an important role in the evolution of tribological system of disc brakes. Due to different wear mechanisms, particles of varying sizes detach from pad and disc surfaces which are crushed and milled down to smaller particles while they are trapped between the contact surfaces [43, 56]. The mixing of these particles and oxidation creates a new type of material called the third body [57]. Hard particles retain their original size whereas soft particles are milled together and finally end up with a very small grain size [38]. Part of the third body is stored in the contact area while the rest is released from the system [58, 38]. Presence of the third body can be related to higher CoF. In [43], the third body was removed by blowing the pad and a reduction in CoF was observed and if the bedding procedure is performed again, which recreates the third body, the CoF would tend to assume high values again. In general it has been argued that due to plowing of the third body CoF increases for a system but it can also decrease if some particles roll and serve as rolling bearings [47].

A third body can consist of all the elements of the friction material and the disc and may also contain their oxides [43, 59, 38]. Furthermore chemical composition strongly depends on the loading conditions during most recent brake applications [60, 43]. Composition of the third body influences the CoF for a given tribological system [61, 43] e.g. diffusion of oxidized iron particles into the friction material affects the CoF of tribological system as shown by Severin and Dörsch [62]. Similarly Kemmer showed that CoF has a correlation to the amount of iron oxide particles in the interface [43]. He also claimed that thickness of the third body has an influence on the frictional performance but in the work by Wirth et al. [61] this was not observed.

Owing to the variety of materials used for friction partners and lack of observation during braking operation as most of the investigations reported in the literature involve examination of contact surfaces after brake operation, there is a lack of agreement between researchers regarding the evolution of the contact interface. There have been mainly two approaches to study the processes taking place at the pad-disc interface [38]. The first approach, presented in [45, 63, 17], relates the brake performance to the formation, growth and degradation of contact plateaus. These contact plateaus, as presented by Eriksson et al. [17], are formed due to removal of less wear resistant material from pad surface which results in direct contact of more wear resistant constituents typically reinforcing ingredients with the disc. Later the area of direct contact can grow by compaction of wear debris (third body) against primary plateaus, forming secondary plateaus as shown schematically in figure 15. These contact plateaus constitute about 10-20% of the apparent pad surface area [45]. It is supposed that main energy conversion happens at these plateaus [37]. The second approach [38, 56, 64, 65, 60] relates the brake performance to the third body layers formed during brake

Figure 15: Schematic representation of the first approach showing the primary and secondary contact plateaus, according to [58, 17].

Figure 16: Schematic representation of second approach showing the third body layer between a pad and a disc, according to [38].

application between pad and disc. This is based on the idea that pad and disc are separated by a layer of third body which consists of fine grained wear debris with some coarser particles as shown in figure 16 [38].

The third body forms different layers at the interface whose thickness mainly depends upon the temperature [60, 65, 56]. These layers are attached to the surfaces of both disc and friction material [57, 60]. Unfortunately, there is no consensus among the authors concerning the nomenclature of third body at the interface. In figure 17

Figure 17: Schematic representation of the third body layers according to [43].

terminology adopted in [43] is described and will be used in the current text. The film that is attached to the disc surface is called transfer film, the film attached to the pad surface is called friction film and the loose particles that remain between these two films is called friction layer. The films should not be considered as continuous but more like regions attached to the surfaces [43, 58, 60]. Third body layers prevent direct contact between the friction partners and reduces wear [66, 42]. A transfer film can fill grooves and wear troughs and make disc surface smoother [57]. A uniform transfer film results in a relatively stable friction behavior at elevated temperatures [42].

3.3 Tribological characteristics of disc-pad system

Contact behavior of the disc-pad interface significantly deviates from most other tribological systems. This system has been studied extensively from microscale to nanoscale. Now some tribological aspects of disc-pad systems which are peculiar to disc brakes will be described. This description is mostly based on the previously published works [45, 63, 17, 38, 56, 37, 67]. In these works friction materials of different compositions were used so some of the phenomenon described here could not be valid for all other materials as also explicitly mentioned in [17, 67].

3.3.1 Bedding

New disc brakes are usually subject to bedding-in procedure. During bedding of a disc brake, moderate brake is applied for a number of cycles so that a stable mean CoF is reached [19, 56]. It is also called running-in, breaking-in, burnishing or conditioning.

During bedding of a new disc, there is an increase of the mean CoF. A virgin disc surface has a spiral ridge pattern resulting from turning operation and when this ridge

Figure 18: Schematic representation of bedding of a disc without showing a third body.

is gradually worn off during bedding, as shown schematically in figure 18 which is motivated by the figure 5.2 in [43], it results in higher CoF due to smoother surface [17]. Figures 4.20 and 4.21 in [43] show the three dimensional profile of disc surface with the help of light interferometry which highlight the smoothening of ridges due to wear.

In the same way bedding of a pad results in an increase of mean CoF as shown schematically in figure 19. In the beginning there is a sharp increase in CoF. It is due to the removal of less wear resistant material from pad surface which results in direct contact of more wear resistant constituent typically reinforcing ingredients with the disc, as shown in figure 20, which usually have higher CoF [17]. This area of direct contact is called a primary contact plateau and stands slightly higher than the less wear resistant constituents after some sliding of the pad against the disc. Later the area of direct contact can grow by compaction of wear debris against primary plateaus, as shown in figure 20, forming secondary plateaus. Formation and growth of secondary plateaus depends upon temperature, humidity, shear forces and normal pressure [45, 17]. On the opposite, secondary plateaus can degrade due to different mechanism e.g. erosion, abrasion or hitting by a surface irregularity in disc surface [45, 17]. Changes in shape and size of secondary plateaus can happen over time due to different growth and degradation mechanism [17]. Slow increase of CoF, after sharp increase in the beginning, can be explained with the slow increase of real contact area due to formation and growth of secondary contact plateaus and flattening of primary contact plateau surface with plastic yielding and wear [17, 56]. At the end of bedding procedure, CoF seems to be stationary, it could be attributed to dynamic equilibrium of patch formation and patch degradation [56].

In the context of third body layer, Österle and Dmitriev [56] explained the low CoF in beginning due to the wear of softest constituent of the pad, mostly graphite, at the outset which forms a film covering primary and secondary contact plateaus. Later when other constituents are worn and mixed with this layer, changing the composition of the third body, it offers relatively higher CoF.

3.3.2 In-stop CoF increase

CoF increases substantially during each individual braking as shown in figure 19. At the start of each braking, CoF is lower than the mean value shown in the graph but at the end is at a higher value. This behavior can be attributed to many factors. As the temperature and pressure increases during a brake, secondary plateau can grow

Figure 19: Schematic representation of evolution of coefficient of friction during bedding-in of pad as shown in [17]. Solid line represents mean coefficient of friction while dashed line represents in-stop friction increase which is shown only for a few braking operations.

Figure 20: Schematic representation of bedding of pad, showing formation of primary and secondary plateaus according to [17, 37]. It also highlights flattening of primary contact plateau surface due to plastic yielding and wear.

Figure 21: Schematic representation of dependence of coefficient of friction on temperature.

significantly because wear debris is more prone to sinter [45]. But when the load is removed at the end of braking, secondary plateaus deteriorate as the patch degradation mechanisms dominate over patch formation mechanisms [45]. Another reason is the speed reduction during braking as the CoF is usually higher at lower sliding velocities [68, 69, 67]. It could also be due to change of pad position during brakings. As the wear on pad and disc is not uniformly distributed [18], so both pad and disc adapt somewhat to each others shape. But the pad is not fixed due to clearance between moving parts, e.g. back plate and caliper, and elastic deformations of different components during operation. So during each braking there is minor change in the position and alignment of the pad which result in a reduced area of real contact in the beginning. This change will result in some wear or deformation so that both pad and disc again adapt to each other [45]. Another reason could be the contamination of contact plateaus e.g. oxidation, while the pad is not in contact with the disc at the end of braking. These oxides could act like lubricants [69] and reduce the CoF initially but when they are removed it would increase the CoF [17]. Österle and Dmitriev noted that this phenomenon can not be explained in the context of third body layer [56].

3.3.3 CoF dependence on temperature, force and velocity

Individual mechanisms of friction are dependent upon temperature, normal load and sliding velocity thus it seems reasonable to assume that CoF is dependent upon these parameters [70]. In many studies [70, 60, 71, 72, 40, 67] CoF was found to be dependent on temperature. Temperature affects the CoF for different friction materials in different ways but a common scenario could be like shown in figure 21. In the figure, initially the CoF increases with increasing temperature and after reaching its peak value starts decreasing. This initial increase and appearance of peak can be attributed to viscoelastic properties of the resin at elevated temperatures [40]. Furthermore as the temperature increases during braking, secondary plateau can grow significantly because wear debris

is more prone to sinter [45] and affecting the CoF positively. Later decrease of the CoF can be attributed to thermal decomposition of the resin, aramid pulp and cashew particles etc. [40, 72, 65]. This thermal decomposition can also lead to loosening and detachment of fibres (primary contact plateaus) [56] which also affects the CoF negatively. In general it could also be said that at higher temperatures, shear strength of materials decreases which leads to a lower CoF [69].

In the context of third body layer, it has been observed that the third body formed at elevated temperature is rather thick (on the contrary, some studies also show decrease in thickness at elevated temperatures [65]) and shows a layered structure [60, 56]. Easy shearing of the different layers along their interfaces explains the low CoF. Another reason for reduced CoF could be the presence of a thin carbonaceous film [60] at the interface which act as a lubricant and support easy shearing between layers [56].

In many studies [70, 67, 59, 69, 42, 73, 52, 74] CoF was found to be dependent on braking force and velocity. In most of these studies CoF shows decreasing trend with increasing velocity while it shows mixed trend with increasing load. An increase in braking force causes the elastic compression of the pad which quickly results in an increased area of real contact for already engaged contact plateaus and also more contact plateaus become engaged [17]. Furthermore an increase of real contact area with the formation and growth of secondary contact plateaus, and flattening of primary contact plateau surface with plastic yielding and wear, also occurs although slowly. This increase of real contact area explains the increase of frictional force with increased braking force. It can be said that intensity of the processes causing the increase of real contact area determine how the CoF behaves with increasing braking force.

In general, decrease of CoF at higher velocities can be explained as: Due to higher velocity there is less time for asperity contact and therefore less time for the deformation of asperities which results in reduction of real area of contact [47]. This reduction of real contact area causes the lower CoF.

3.3.4 CoF hysteresis

CoF is slightly lower during pressure increase than in steady state at a given pressure. Correspondingly CoF is slightly higher during pressure decrease than in steady state at a given pressure. This behavior of CoF constitutes friction hysteresis [17] and shown schematically in figure 22. This behavior could be attributed to relatively slow adaptation of the tribological interface, e.g. size of secondary contact plateaus, to the prevailing conditions [17]. As different pads have different plateau formation and degradation behaviors so they also show different hysteresis behaviors [68].

3.3.5 Dependence of tribological behavior on history

It has been discussed above that after a history of brake application, surfaces of friction partners are different than a virgin brake system. Furthermore a third body is generated and forms different layers at the interface. The chemical composition and thickness of layers strongly depends on the loading conditions during most recent brake applications [60, 43, 65, 56]. So it can be said that history of a tribological system determines its condition.

It has also been discussed above that size of the contact plateaus and smoothness of their surfaces influences the CoF of a brake system. It has been shown that composition of a third body influences the CoF for a given tribological system [61, 62, 43]. It is

Figure 22: Schematic representation of hysteresis of coefficient of friction, according to [45].

Figure 23: History dependence of the tribological behavior according to [43].

also claimed by some researchers that thickness of a third body has an influence on the frictional performance [43, 56]. So it can be said that condition of a tribological system determines its behavior and as the condition is determined by the history, it can be concluded that history of a tribological system determines its behavior [43] as shown schematically in figure 23.

3.3.6 Inertia of CoF

After a change in loading conditions, CoF will change to new conditions only after a characteristic time period [37]. This behavior is known as inertia of μ . This could

Figure 24: Schematic representation of dependence of wear rate on temperature.

be explained with slow adaptation of the composition of the third body with changing loading conditions. The CoF for a given system depends on composition of the third body [43] which by itself depends on the loading conditions [60]. The composition does not adapt instantaneously with the changing conditions instead takes some time to alter. The time required for the change of composition results in inertia of μ .

3.3.7 Wear

In several studies of brake system [66, 40, 60] wear rate was found to increase with increasing temperature. For a friction material with phenolic resin as binder, the wear rate below a critical temperature shows a slow increase, but above it the wear rate increases rapidly as shown schematically in figure 24. Furthermore below the critical temperature binder plays a minor role in the wear resistance of the friction material, but above it the wear rate is strongly influenced by the thermal decomposition of the resin [40].

In a few studies [74, 73] wear rate was found to decrease with increasing load. This could be due to higher exposure of contact plateaus due to wearing of less wear resistant constituents [74] and growth of secondary plateaus with increasing pressure which eventually results in less wear rate. In some other studies [52, 74] wear rate was found to decrease with increasing velocity.

In a study, it was shown that wear rate of friction material and friction coefficient are not related [61]. In another study, Cho et al. [65] observed that there is no conclusive correlation between wear rate and third body layer thickness.

3.3.8 Miscellaneous

If the temperature of cast iron disc exceeds 700°C and subsequently cooled rapidly, material can transform its phase to martensite [75, 76, 77]. Martensite has a different CoF with the pad material than pearlite [77].

3.4 Models for friction phenomenon in brakes

To numerically simulate the behavior of disc brakes realistically, it is important to represent the frictional behavior at the contact interface accurately. Complexity of tribological interface and interaction between different parameters make the determination of reliable and exact friction laws difficult [67]. Unfortunately many researchers use classic laws of friction in their sophisticated finite element analysis which is not sufficient. Now some friction models will be described which have been specifically developed for brakes. All the notations used to describe these friction models have been changed to be consistent in this paper.

Rhee [70] investigated the frictional properties of brakes for a semi-metallic friction material sliding against cast iron. He observed the frictional force to be dependent on the normal load, sliding speed and temperature. He found the Amontons' law unsatisfactory to describe the friction behavior and instead proposed the the following equation.

$$F_r = \mu F^a v^b \quad \text{at} \quad T_i, \quad (4)$$

where F_r is the friction force, F the applied normal load, v the sliding speed, and a and b are constants. In this model, μ is constant regardless of the load and speed but dependent on temperature, and a and b are also dependent upon temperature.

Thuresson [78] introduced the following equation to describe the correlation between CoF and temperature.

$$\mu(T) = \mu_0 - Tb, \quad (5)$$

where μ_0 is the CoF at the reference temperature T_0 and b is a constant which can be determined by experiments.

Saffar et al. [52] introduced the following equation to describe the correlation between CoF and temperature.

$$\mu = \mu_0 + k(T - T_0) \exp\left(-\frac{E}{RT}\right), \quad (6)$$

where T_0 is a reference temperature, μ_0 the CoF at the reference temperature, R the universal gas constant, k a constant and E represents the activation energy that varies with velocity. The parameters of this model can be obtained with the curve fitting procedure based on the experimentally determined CoF vs. temperature graph.

Kemmer [43] performed experiments to find if the CoF depends upon sliding speed, temperature and pressure. He obtained the following expressions for two different pad materials by using multi-variable regression.

$$\mu = 0.234 + (8.5p - 5.15v + 3.05T)10^{-4} \quad (7a)$$

$$\mu = 0.334 + (-13.9p - 10.4v + 3.9T)10^{-4}, \quad (7b)$$

where p is the brake line pressure in [bar], v the vehicle velocity in [km/h] and T is the temperature in [°C]. A number of brake applications were performed to obtain data for determination of these relations. A bedding procedure was performed before each brake application to bring the tribological system to a comparable condition as a brake application might modify the interface. Kemmer noted that such expressions are

neither general nor universal and need to be determined for a given disc-pad system and operation range.

Ostermeyer and Müller [37] proposed a dynamic friction law taking into account the interconnection of the friction and temperature dynamics. It is a physically derived law based on the concept that tribological behavior is determined by growth and degradation of the contact plateaus. This is expressed by the following differential equations.

$$\dot{\mu} = -\alpha((\beta + x(t))\mu - \gamma T_P) \quad (8a)$$

$$\dot{T}_P = \delta(T_P - T_0 - \varepsilon x(t)), \quad (8b)$$

where $\mu = \mu(t)$ is current CoF, $x(t)$ is the product of current normal force and current velocity, T_P is the temperature of the patches, T_0 is the integral disc temperature, α is the time constant for the CoF, β is the system parameter representing the correlation between the patch growth and the existing patch area, γ the system parameter representing the correlation between the patch temperature and the patch growth, ε is the system parameter representing the correlation between the friction power and the generated heat on the patch, and δ is the time constant for the patch temperature. It was shown that with the proper selection of the parameters in these equations, different phenomenon e.g. fading, inertia, hysteresis and in-stop friction increase can be reproduced [37, 79].

3.5 Models for wear in brakes

In the studies of braking systems, Archard's wear equation (e.g. in [80, 78, 81] and Rhee's wear equation (e.g. in [25]) have been used to model wear.

To account for temperature dependency of the wear rate for brakes, Thuresson [78] assumed $k = k(T)$ while adopting the Archard's wear law (equation 3) and introduced the following model for computing $k(T)$

$$k(T) = k_0[1 + Tc_1 + H(T - T_t)c_2(e^{c_3(T-T_t)} - 1)], \quad (9)$$

where k_0 is the wear coefficient at a reference temperature, $H(\cdot)$ the Heaviside function, T_t the temperature from which wear is assumed to grow exponentially, and c_1 , c_2 and c_3 are constants. These constants can be determined with experimental data. This model was also used in the work by Vernersson and Lundén [80].

Rhee [82] proposed an equation for predicting material loss of friction material. The equation is given as

$$\Delta W = kF^a v^b t^c, \quad (10)$$

where ΔW represents the weight loss of the friction material, k is the wear factor, F the load, v the sliding speed, t the time, and a , b and c are constants for a given system.

3.6 Discussion

To numerically simulate the behavior of a brake system with sufficient accuracy it is important to have a friction model which is valid pointwise. In the literature mostly or almost all reported friction data is the global value of CoF for a given experimental setup and is not valid pointwise throughout the contact region as also noted by [3].

Then question arises if the different global phenomena observed for CoF, as described in the section 3.3, are a result of variation of local CoF or due to other changes, e.g. contact pressure distribution, happening at the contact interface. Severin and Dörsch [62] observed that local temperatures of contact surface changes periodically during brake application despite constant load conditions. So they concluded that the local pressure or the local CoF or both also undergo a periodic change. In a work by Heussaff et al. [67], local or pointwise CoF is reported for a pin-on-disc tests. First they determined a global frictional behavior by using data from the tests. Then with the help of finite element simulations and gradient based optimization they determined local friction laws depending on the contact pressure, velocity and temperature. This seems to be the only work available in the literature concerning the determination of local CoF. It seems that determination of the local CoF and its dependence on different state variables will be a big challenge for the researchers in the future as accuracy of numerical simulations to predict the behavior of brakes depends on the CoF between friction partners.

Similarly wear models are required which represent realistic behavior pointwise as wear also plays important role in evolution of the contact interface and hence its tribological characteristics.

Sometimes there are different opinions in the literature among authors regarding the contact interface, the obvious reasons are the difference of materials for friction partners for each test and different experimental setups. Another reason could be loss of the third body when pad and disc are separated for analysis purposes as mentioned in [38]. Most of the experimental works about the disc-pad interface describe the disc or pad surface after the brake application. Only a few papers [38, 63] are about the situation during braking.

4 Operational issues

Now some of the issues related to disc brakes encountered during brake application will be described. First brake fade will be described which is a very common phenomenon. A separate section is devoted to geometrical deviations which do not present a major problem by themselves but a source of vibrations. The vibrations originating at a disc-pad interface are covered in a separate section.

4.1 Brake fade

Brake fade is a term used to describe the decrease in brake effectiveness or brake power. It is usually caused due to excessive heat in the tribological system. It could be divided into four sub-categories:

μ -fade

Decrease of the CoF during a brake application is called μ -fade. In general it could be argued that the CoF should decrease with increasing temperature as the resistance of asperities to elastic and plastic deformation for contacting materials is reduced with higher temperatures. Rhee [70] observed that CoF decreases with the increase of normal load, speed and temperature. So he described that fade consists of three components: load fade, speed fade and temperature fade. The mechanisms influencing the μ with the change in force, speed and temperature have already been discussed in section 3.3.3.

Gas fade

Decrease of frictional force due to build up of gases at the interface is called gas fade. Herring [83] showed that as the temperature rises, gases evolve from the friction material mainly due to thermal deterioration of the resin. The gases at the interface cause buildup of pressure resulting in the reduced resultant brake force which in turn causes the reduction in frictional force. The frictional force can recover even at high temperature which can be attributed to the decrease in gas generation rate due to the resin approaching complete decomposition and increase in gas venting due to excessive wear at the high temperature. Another mechanism [2, 83] causing fade when high gas flow rate occur is that a thin layer of hot gas between the friction partners forms and contact area reduces significantly. A slot in the pad surface helps the gas to escape and avoid fade.

One way to reduce fade is to remove the heat as quickly as possible from the interface. The friction materials containing copper fibres exhibit better fade resistance partially due to high thermal conductivity of copper [42]. To boost the thermal conductivity and increase the fade resistance, brass and copper powders are also incorporated in the friction materials [84].

Brake fluid fade

If the brake fluid temperature reaches its boiling point then it can form vapors in brake line [2]. This can happen if too much heat enters the hydraulic system in case of consecutive hard brakings or a prolonged braking. As a result brake effectiveness is reduced which is termed as brake fluid fade.

Water fade

In a rainy or humid environment, water might get trapped between disc and pad surfaces. As a result brake effectiveness is reduced which is termed as water fade [2]. In wet environment CoF decreases significantly [69, 73] which could be a serious issue in rainy conditions. The decrease in CoF is significant with increasing sliding speed which is attributed to the hydrodynamic effects of the water films which are significant at higher speeds and at low brake force [69].

4.2 Geometrical deviations of disc and pad

There could be many sources of geometrical deviations or irregularities in a disc and a pad. Some of those are permanent and some are temporary. Here permanent deviations are those which remain after the part has returned to ambient temperature and temporary are those which disappear after returning to ambient temperature. Now some of the major deviations will be described. But it is important to mention here that different deviations may superimpose and lead to very complex dynamic geometries [85].

4.2.1 Coning and buckling of disc

As a result of heat generation at the interface, in addition to local changes of the contact surfaces there are global deformations occurring in disc and pad. Due to different geometries of discs each presents different geometrical constraints to the

Figure 25: Schematic representation of coning and buckling of a brake disc.

Figure 26: Cross-section of a simplified disc, highlighting the coning of a disc due to geometrical constraints.

thermal expansion. So the deformations can appear in different forms in different discs. Some of the most commonly observed thermal deformations are coning and buckling as shown in figure 25 [77, 85, 9, 86, 76].

Generally, disc coning occurs due to the different thermal expansions of the outboard and inboard cheeks as the outboard cheek is integrally connected to the mounting bell which represents a constraint for its expansion as shown in figure 26 [8].

In a study on thin discs by Davis et al. [87] it was found that if the disc is heated to a critical average temperature buckling will occur. Buckling of a brake disc can be explained by the occurrence of relatively higher temperatures near the periphery of the

Figure 27: Schematic representation of different wavelengths for a brake disc.

disc as compared to near the mounting bell. This results in relatively higher expansion of the material near periphery both in radial and circumferential directions. But as the expansion is constrained in circumferential direction due to closed shaped of the disc so it buckles to accommodate the expanding material. Furthermore, a thermal moment is generated if the temperatures are non-symmetric with respect to midplane of the disc, which could cause buckling [88]. This thermal moment vanishes if heat generation and disc geometry are symmetric about the midplane. The wavelength of the corrugations can vary for a given disc as shown in figure 27. It depends on many factors, one of them is the thermal gradient between the friction ring and mounting bell [85]. Panier et al. [76] suggested that it could also depend on the ratio of the disc mean perimeter to the contact length between the pad and the disc. If the temperatures are not too high then only elastic strains occur during buckling and disc will return to its previous shape (ignoring wear) after it is cooled to ambient temperature. But if temperatures are above a critical limit, plastic yielding may occur and then disc will be susceptible to buckling in the future at the same location [76, 88], i.e. troughs and crests will occur at the same locations.

4.2.2 *Disc runout*

Ideally a disc friction surface should be perfectly flat and lie normal to the axis of rotation but there might be deviations from the ideal situation, which is termed as runout or more specifically axial runout. It is usually measured by noting the difference between highest and lowest points on a disc surface. Figure 28 shows the condition of runout of a disc schematically. Commercial vehicle discs can show runout even in as-

Figure 28: Schematic representation of disc thickness variation (right) and disc runout (left and middle).

manufactured condition which could be due to inadequate stress relieving of the casting [77]. Furthermore, buckling of a disc, as described in the previous section, will also appear as runout. One reason of the runout could be the warpage of disc due to uneven tightening of the bolts during installation. Another reason is the presence of dust or any other contamination at the mounting surface during the fitting of the disc. Permanent distortions can also happen when residual stresses in the disc as a result of casting process are relieved at high disc temperatures [85]. In the studies by Spooner et al. [89] and Shin et al. [90] it was shown, by using the neutron diffraction technique, that average residual stresses are reduced after heat treatment of a cast iron disc. Shin et al. [90] also showed that a non heat-treated disc when exposed to a high temperature experiences run out which is more than twice the run out experienced by a heat-treated disc. In another study, Hofwing [91] showed with numerical simulations that residual stresses exist after casting and machining of the disc.

4.2.3 Disc thickness variation

Ideally a disc should have same thickness all over but in reality a variation in thickness can occur which is termed as disc thickness variation (DTV). Figure 28 shows the DTV of a disc schematically. There could be many reasons for DTV.

Commercial vehicle discs can show DTV even in as-manufactured condition [77]. Furthermore after a disc has developed a runout, there would be some areas where pad will not be able to make contact and at some other areas contact pressure will be more than average. Due to this high contact pressure more heat is generated and as a result material in these areas shows more expansion. This results in DTV which could disappear as the disc cools down and returns to its original shape. At the same time wear plays its role and removes material from the disc surface where contact pressure and temperature are relatively higher which causes permanent DTV. Furthermore if the temperature of cast iron disc exceeds 700°C (738°C according to [85]) and subsequently cooled rapidly, material can transform its phase to martensite [75, 76, 77]. Due to relatively higher volume of martensite compared to pearlite, the disc can grow locally

Figure 29: Simplified representation of the convex bending of a pad.

which leads to DTV and remains even after cooling. Development of an uneven transfer film on the disc surface can also contribute to the DTV.

4.2.4 Thermal deformation of pad

Global thermal deformation of pad is usually referred as convex bending in literature [18, 17, 92] and results in reduced area of contact in the middle of the pad as shown in figure 29. The frictional heat generation at the interface results in higher temperature of the pad surface in a short period of time as compared to the inner region of the pad and the support plate. Consequently, the surface expands more than the inner region of the pad and the support plate which results in the convex bending.

4.2.5 Cracks on disc surface

Due to the severe working conditions during operation, macro-cracks might develop on the disc surface in the radial direction [93, 94]. It has been shown in many previous works, e.g. [86, 95, 96, 97], that during a hard braking, high compressive stresses are generated in the circumferential direction on the disc surface which cause plastic yielding. But when the disc cools down, these compressive stresses transform to tensile stresses. For repeated braking when this kind of stress-strain history is repeated, stress cycles with high amplitudes are developed which might generate low cycle fatigue cracks after a few braking cycles. Dufrénoy and Weichert [86], confirmed the existence of residual tensile stresses on the disc surface by measuring with the hole drilling strain gage method.

4.3 Vibrations and noise

Not all the kinetic energy is converted into heat during friction braking, part of it might be dissipated in the form of vibrations. These vibrations are a major comfort problem.

A lot of literature has been published related to the subject of brake noise and vibrations. Several review articles [98, 99, 100, 101, 102, 103, 3, 104, 105] cover the published literature in detail. Latest reviews of the vehicle noise and vibrations, in general, can be found e.g. in [106, 107].

Now a classification of these vibrations as a function of frequency will be described.

4.3.1 Judder

Judder is a low frequency forced structural vibration due to excitation generated at disc-pad interface [29]. Its associated airborne noise is called hum. It is a forced vibration mostly due to geometrical deviations of disc and variations of CoF. Its frequency is proportional to the wheel speed and hence judder frequency is usually mentioned as a

multiple of wheel speed e.g. second-order judder means the judder frequency which is twice the revolutions per second of the wheel. The upper limit of its frequency depends on the maximum wheel speed and judder order (usually < 1000 [Hz]). Judder vibrations are transmitted to the vehicle body and can be felt by driver in the brake pedal, steering wheel or floor.

Hot judder

Hot judder, also called thermal judder, can be defined as the judder due to thermal cause. In other words it is generated due to temporary geometrical deviations of a disc as temperature goes higher and will disappear with disappearance of geometrical deviations as the disc cools down. It is caused due to high thermal input to the brake in a short period of time that results in a thermoelastic deformation, and eventually thermoelastic instabilities in the form of hot spots [108].

Cold judder

It occurs due to permanent geometrical irregularities of the disc. So it can happen even at the start of brake application when the disc is cold.

Normally judder consists of both hot and cold judder [29]. So it is difficult to separate them and hence these classification could cause confusion.

4.3.2 Groan

Groan is a low frequency structural vibration. Its associated airborne noise is called moan. It occurs primarily as a result of friction characteristics of friction pair. Its frequency is generally independent of wheel speed and temperature [19]. It is usually divided in two categories, creep groan and dynamic groan. Creep groan is related to stick-slip motion and caused by a higher dynamic CoF than static CoF. Dynamic groan is an instability phenomenon as a result of friction-velocity characteristic known as negative damping.

4.3.3 Squeal

Squeal is a high frequency (> 1000 [Hz] [3]) vibration of the brake components during a braking action, primarily caused by resonance [29]. It results in a noise which is a nuisance to passers-by and vehicle occupants. The vibrations propagate through the air rather than through the vehicle structure [29]. The most significant complication in brake squeal research is its fugitive nature, i.e. brake squeal can sometimes be non-repeatable [103].

A braking system can squeal at a number of distinct frequencies which often correspond to the natural frequencies of brake system components [3]. More precisely, squeal occurs in well-determined narrow frequency ranges because of changing conditions, e.g. temperature and pressure, may change the modal behavior of the brake components [24]. These frequencies are independent of the speed of the vehicle but remain constant for a given brake system [24].

Brake squeal is a complex phenomenon. There has been significant progress towards understanding the mechanism causing the generation of brake squeal but there is yet a lack of complete understanding of the causes of squeal noise [109]. Today, it is

commonly accepted in the disc brake community that squeal is initiated by an instability due to the friction forces leading to self-excited vibrations [110]. Several explanations have been proposed about the onset of instability. These could be categorized as

1. Decreasing μ_k with increasing v_t

It has been proposed that a necessary condition for the onset of instability is the decrease of μ_k with increasing v_t . This school of thought is based on the work of Mills [111].

2. Sprag-slip

It proposes that the squeal occurs due to unstable oscillation in the system which arises with the variation of friction force as a result of variation of the normal force. So this theory can be applied even when CoF is constant. This school of thought is based on the work of Spurr [112] and has been extended in other works e.g. [113, 114, 115].

Historically, to simplify the modelling of disc brakes, contact forces have not been considered as a moving load due to the low rotational speed of disc [104]. This moving load acts at different locations at different times which could cause an instability resulting in squeal even if the force itself is constant. Ouyang et al. considered the contact forces as a moving load in many of their works [116, 117, 118]. Cao et al. [119] modelled the brake squeal as a moving load problem and predicted unstable frequencies which were in a good agreement with the experimentally found frequencies.

3. Modal coupling

It proposes that squeal is the consequence of the unstable resonances induced by the friction force between disc and pads. This theory suggests that squeal is driven by coupling of modes of the brake system components that are close in frequency. This explanation is based on the work of North [120, 98].

4. Hammering

It proposes that repeated impacts of a pad on a disc can excite it into a state of vibration. These impacts could be as a result of rocking of pads due to geometrical deviations of a disc. This explanation was proposed by Rhee et al. [121]. It is reported in [3] that sanding the surfaces of a disc is considered a remedy of brake squeal which could be explained with the hammering theory as sanding reduces the geometrical deviations and makes surface smoother. It is also reported in [122] that pads with many small plateaus generate more squeal than do pads with few large plateaus, which could be related to hammering theory [3].

Nowadays decreasing μ_k with increasing v_t and sprag-slip are no more considered the actual mechanism exciting the squeal [3, 109].

A considerable amount of literature has been published on brake squeal. Researchers have used different experimental and computational techniques to understand the phenomenon of squeal. Early attempts to understand the phenomenon of squeal using computational techniques has been through the development of few degrees of freedom models, called minimal models or lumped parameter models. These models have been reviewed in [3, 110, 105]. Recently finite element method, which results in models with

high number of degrees of freedom, has become popular. In contrast to minimal models, the finite element method has the advantage for accurate representation of the complex geometries and boundary conditions. One of the earlier use of finite element method was to determine the modes and natural frequencies of brake discs [3].

Simulation and analysis methods are divided in two main categories: complex eigenvalue analysis and transient analysis [104]. The complex eigenvalue analysis studies the brake system at static steady sliding states and is mainly used to predict squeal frequencies. This was initially used on minimal models and later it was extended to finite element models [123]. A transient analysis is used to determine self-excited vibrations during the squeal event [124]. A transient analysis can take into account the time-dependent loads and other non-linearities. Complex eigenvalue analysis has been used by many authors to predict the onset of instability which is considered the cause of squeal [125]. During the squeal vibrations, nonlinear effects can not be ignored and hence a transient analysis is required. It is reported in [124] that during transient vibrations, an additional unstable mode can appear which is not predicted by the complex eigenvalue analysis.

Study of brake squeal requires the consideration of many complex phenomena and parameters e.g. contact evolution at disc-pad interface, wear, material properties which are mostly temperature dependent and load/boundary conditions etc. Furthermore due to manufacturing tolerances/variations of different parts could show inconsistent characteristics. This complexity presents a major difficulty for investigation of brake squeal problem as repeatability of measurements during an experimental study becomes a major issue [109]. Modelling of such a complex system and correlating the computed results to the measurements then becomes a challenging task. There is no validated predictive model of friction induced vibration to date [126]. It has been suggested that a brake system is highly sensitive to uncertainties [127]. The study of sensitivity in disc-pad systems is now attracting the attention of research community (see e.g. [128, 129, 126, 130, 127]). In [131], to take the uncertainties into account, descriptive statistical analysis tools were used. In [132], brake squeal was treated as a chaotic phenomenon.

In an effort to avoid the complexity of a commercial brake system, researchers have also developed the simplified test rigs. Less complicated geometry of a simplified set-up also requires a simple model which ultimately reduces the uncertainties. Simplified set-ups can be classified into pin-on-disc systems (e.g. [133, 134]), beam-on-disc (e.g. [135, 136]), laboratory brake (e.g. [137, 138]) and tribo brake (e.g. [125, 139]). These set-ups (except pin-on-disc) are reviewed and described in [109]. A chronological overview of simplified set-ups is given in [132]. In the opinion of Akay et al. [109], the advances towards the understanding of underlying mechanism of squeal has been mostly due to the simplified set-ups as the repeatable squeal conditions become possible and in parallel finite element models verified these results, at least qualitatively.

Structural damping has been shown to be of primary importance to the stability of mode couplings [140, 141, 142, 143]. Effects of damping have been considered on minimal models in many studies (see e.g. [144, 140, 141, 145]) but mostly ignored in finite element models due to complexity [123]. Fritz et al. [123] developed a finite element model of a brake system and showed that damping does not always stabilizes the brake system instead it could also make the system unstable depending upon the modes involved.

In their review, Kinkaid et al. [3] describe that it is a common belief that, to avoid squeal, the brake pads, disc and caliper are designed in a way that their natural frequencies in the audible range are as isolated as possible. However they also mention that this is not sufficient as there are cases reported in literature where squealing frequencies depend only on the disc brake rotor and are independent of other components in a brake system. It was also demonstrated in [146] that effects of structural damping on mode coupling instability must be taken into account to avoid design errors. Structural modifications of components of a disc brake system have been used by many researchers to change their modal characteristics. Many disc modifications e.g. cheek thickness, vanes shape and size, and drilled holes have been proposed to suppress squeal [147]. Modification of brake pads such as chamfers and slots can also help to reduce squeal noise [14, 15, 16]. It was explained in [148] that chamfering could modify the merging behaviour of the modes and thus reduce squeal. It has been proposed that breaking the symmetry of brake disc could be a countermeasure to squeal [149, 150, 151]. In [151] it is shown that asymmetry of a disc results in splitting the double modes and has a stabilizing effect. This knowledge has been used in [152, 153, 154] for optimization of the brake rotor to avoid squeal.

Another solution to suppress squeal is the use of laminated shims which are attached to the back plate of brake pad. These shims increase system modal damping which reduces the propensity for squeal noise [24].

4.3.4 *Wire brush*

It is a superposition of high frequency oscillation with randomly varying amplitude [19]. It is also called roughness noise. It is a structural-borne noise and not the air-borne [155].

5 Conclusions

Disc brake is a complex system and understanding different issues related to its design and operation require expertise from different disciplines e.g. tribology, material science, fluid dynamics, vibrations etc. Disc brakes have evolved a lot over the decades due to extensive research and development. There are still many phenomenon which are not understood fully.

For realistic computational analysis of disc brake systems, it is crucial to further develop non-linear finite element models which could simulate realistic evolution of contact interface. Such models should be capable, atleast, to incorporate a realistic friction and wear model, and temperature dependent material properties.

One of the issue which has attracted a lot of attention of research community is the brake squeal. Due to continuous development of disc brake systems, they have become quieter although squeal problem has not been solved well [3, 119, 132]. The problem of predicting the brake squeal propensity remains a challenging task for brake research community [131].

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