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Sequential Simulation of Thermal Stresses in Disc Brakes for Repeated Braking

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Abstract

In this paper an efficient sequential approach for simulating thermal stresses in brake discs for repeated braking is presented. First a frictional heat analysis is performed by using an Eulerian formulation of the disc. Then, by using the temperature history from the first step of the sequence, a plasticity analysis with temperature dependent material data is performed in order to determine the corresponding thermal stresses. Three-dimensional geometries of a disc and a pad to a heavy truck are considered in the numerical simulations. The contact forces are computed at each time step taking the thermal deformations of the disc and pad into account. In such manner the frictional heat power distribution will also be updated in each time step, which in turn will influence the development of heat bands. The plasticity model is taken to be the von Mises yield criterion with linear kinematic hardening, where both the hardening and the yield limit are temperature dependent. The results show that during hard braking high compressive stresses are generated on the disc surface in the circumferential direction which cause yielding. But when the disc cools down, these compressive stresses transform to tensile residual stresses. For repeated hard braking when this kind of stress history is repeated, we also show that stress cycles with high amplitudes are developed which might generate low cycle fatigue cracks after a few braking cycles.

Keywords: Eulerian framework, frictional heat, thermal stresses, disc brake, repeated braking

1 Introduction

Disc brakes are an important component of a vehicle retardation system. They decelerate a vehicle by converting its kinetic energy into heat. During braking, a set of pads is pressed against a rotating disc and due to friction, heat is generated at the disc-pad interface and causes the disc surface temperature to rise in a short period of time. This heat ultimately transfers to

the vehicle and environment and the disc cools down. Due to these severe working conditions during operation, macrocracks might develop on the disc surface in the radial direction. It has been shown in many previous works, e.g. [1] and [2], that during a brake application, compressive stresses are generated on the disc surface which cause yielding of the material. Then, when the disc is cooling, residual tensile stresses develop. For repeated hard braking of heavy trucks, microcracks in the radial direction of the disc brake might develop after a low number of cycles. This low cycle fatigue (LCF) phenomenon might be explained by the stress amplitudes developed during the brake cycles. In this work, we propose an efficient sequential approach for simulating these kind of stress amplitudes.

In previous works, different numerical approaches have been used to predict the thermomechanical behavior of disc brakes. Richmond et al. [3] implemented an axisymmetric fully coupled thermomechanical model. They also developed a special finite element for the interface between the friction material and the disc. Kao et al. [4] developed a three-dimensional (3D) FE model capable of performing fully coupled thermomechanical analysis. They used this model to study hot judder in a disc brake. Koetniyom et al. [5] performed sequentially coupled thermomechanical finite element analysis of disc brakes under repeated braking conditions. They considered only a small segment of the disc taking the cyclic symmetry into account and assumed a uniform heat flux. They developed and implemented a temperature dependent material model that allows different yield properties of cast iron in tension and compression. Choi and Lee [6] developed a FE model for an axisymmetric fully coupled thermoelastic contact problem. They used this model to investigate thermoelastic instability in disc brakes. In [1], Dufrénoy and Weichert developed an uncoupled 3D FE model. They simulated only one-twelfth of the disc by considering the axial and rotational symmetries of the disc and used temperature dependent material data. They confirmed the existence of residual tensile stresses on the disc surface by measuring with the hole drilling strain gage method. They also performed cyclic load analysis for repeated braking by implementing a temperature dependent linear kinematic model. Gao et al. [2] developed a fully coupled 3D thermomechanical FE model to investigate the source of fatigue fracture in the disc brakes. They assumed that thermal properties of the materials for disc and pad are invariant with temperature. Kim et al. [7] performed fatigue life assessment for brake discs of railway vehicles. For this they performed fatigue test of the disc material and determined stress-life curves. Then for the computed stresses in the disc brake, they assessed the life of disc by using the Miner's rule. Šamec et al. [8] studied the LCF behavior of nodular cast iron used for disc brakes. They performed strain controlled LCF tests at different temperatures and determined strain-life curves and different LCF parameters.

Today, the usual way to simulate frictional heating of disc brakes in

commercial softwares is to use the Lagrangian approach in which the finite element mesh of a disc rotates relative to a brake pad. Although this approach works well, it is not feasible due to extremely long computational times. For simulating repeated braking this approach is of little importance for practical use. The rotational symmetry of the disc makes it possible to model it using an Eulerian approach, in which the finite element mesh of the disc does not rotate relative to the brake pad but the material flows through the mesh. This requires significantly low computational time as compared to the Lagrangian approach. Nguyen et al. [9] developed an Eulerian algorithm for sequentially coupled thermal mechanical analysis of a solid disc brake. First they performed a 3D contact calculation to determine the distribution of the pressure. Then a sequentially coupled analysis is implemented by first performing a transient heat transfer Eulerian analysis followed by a steady-state mechanical analysis. Recently, Strömberg [10] developed a finite element approach using an Eulerian framework for simulation of frictional heating in sliding contacts. In his approach, the fully coupled problem is decoupled in one mechanical contact problem and a frictional heat problem. For each time step the thermoelastic contact problem is first solved for the temperature field from the previous time step. Then, the heat transfer problem is solved for the corresponding frictional power. In another paper [11] this approach was implemented for simulating frictional heating in disc-pad systems.

In this work, thermal stresses in ventilated disc brakes for repeated braking are simulated by implementing a sequential approach. This approach requires significantly lower computational resources so it becomes feasible to simulate multiple brake applications. First a toolbox developed by Strömberg, which is based and described in his earlier work [11], is used to perform the frictional heat analysis. In this Eulerian approach the contact pressure is not constant, but varies at each time step taking into account the thermomechanical deformations of the disc and the pad. This updated contact pressure information is used to compute heat generation and flow to the contacting bodies at each time step. In such manner, the nodal temperatures are updated accurately and their history is recorded at each time step. Then a script is used to write temperature histories for multiple brake applications into an output file for subsequent use. Because the finite element mesh of the disc does not rotate relative to the pad, the contact region is always well defined and a node-to-node based approach can be adopted. This allows the mesh to be refined only in the region where the brake pad is in contact with the disc, which results in lower computational time. The output file with temperature history is then used in a sequentially coupled stress analysis performed with a temperature dependent von Mises plasticity model with temperature dependent linear kinematic hardening by using Abaqus/Standard.

The results show the presence of residual tensile stresses in the circum-

Figure 1: The workflow of the sequential approach to determine the thermal stresses.

ferential direction on the disc surface after it cools down. For the repeated braking an approximately stable stress-strain loop is obtained. The sequential approach has proved tremendously cheap in terms of computational time when compared to a fully coupled Lagrangian approach. This makes it a feasible technique for simulating repeating braking. This is demonstrated by presenting numerical results.

The outline of the paper is as follows: in Section 2, the frictional heat analysis is described as well as how the corresponding temperature history is exported to the stress analysis. In Section 3, the governing equations for the stress analysis are given. In Section 4, the material properties of all the parts considered during the simulation are described. In Section 5, numerical results are shown for repeated braking of a disc brake to a heavy truck. Finally, in Section 6, some concluding remarks are given.

2 Frictional heat analysis

The workflow of the sequential approach used for simulation of thermal stresses is shown in Fig. 1. An input file, which contains the meshed geometry with appropriate boundary conditions and loads is required for the frictional heat analysis. During this analysis linear thermo-elasticity is adopted and the problem is decoupled in two parts. In the first part, for a given

Figure 2: The sequential approach used during the frictional heat analysis to determine the temperature history.

temperature distribution the contact problem is solved to obtain the nodal displacements and the contact pressure distribution. The contact conditions are treated by applying the augmented Lagrangian approach, where the corresponding equation system is solved by using Newton's method. In the second part, for the obtained contact pressure distribution the energy balance is solved and new nodal temperatures are determined. The heat balance of the disc is formulated in an Eulerian framework, where the convection is governed by the angular velocity of the disc. In order to stabilize the solution as a consequence of the non-symmetry of the convection matrix, we adopt the streamline-upwind approach. The equation systems defining the contact problem and the heat balance are then solved sequentially and a temperature history is generated. In this procedure, the nodal temperatures determined at each time step are taken into account in the next time step to calculate the thermo-elastic deformations of the disc and pad, which in turn will influence the contact analysis. This is shown schematically in Fig. 2. More details about the frictional heat analysis can be found in Strömberg [11].

After convergence in the frictional heat analysis procedure, the nodal temperature history is written to an output file (called ODB file) by using a script. The script exports temperature histories for multiple brake applications into a single ODB file. This output file is then imported into Abaqus/Standard for performing sequentially coupled stress analysis. In this work, a von Mises plasticity model with temperature dependent linear kinematic hardening is used in the latter analysis. Details about this are presented in the next section.

3 Stress analysis

Permanent residual stresses might develop in a component even after the external causes, e.g. heat gradients or external forces, have been removed. In the case of thermal stresses, if the stress levels are sufficiently large to cause yielding in a component, then these may transform to residual stresses when the component cools down. Such residual stresses may affect the behavior and life of a component. In this work, we study such residual stresses in a disc brake to a heavy truck by performing stress analysis as outlined below.

Given the temperature history from the frictional heat analysis, thermal strains are determined according to

$$\boldsymbol{\epsilon}^t = \alpha(T)(T - T_{\text{ref}}) - \alpha(T_i)(T_i - T_{\text{ref}}), \quad (1)$$

where $\alpha(T)$ is the thermal dilatation coefficient, T_{ref} is a reference temperature and T_i is the initial temperature. The infinitesimal strain $\boldsymbol{\epsilon}$ is split into elastic, plastic and thermal strains, expressed as

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^e + \boldsymbol{\epsilon}^p + \boldsymbol{\epsilon}^t, \quad (2)$$

where $\boldsymbol{\epsilon}^e$ and $\boldsymbol{\epsilon}^p$ represent elastic and plastic strains, respectively. The stresses are computed by using Hooke's law as

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\epsilon}^e, \quad (3)$$

where $\mathbf{D}$ is the elasticity tensor. The stresses satisfy the following equilibrium equation:

$$\text{div}(\boldsymbol{\sigma}) = \mathbf{0}. \quad (4)$$

During the stress simulation, the von Mises plasticity model with temperature dependent linear kinematic hardening model is used. According to this, the yield surface is defined as

$$f(\boldsymbol{\eta}, T) = \sqrt{\frac{3}{2}\boldsymbol{\eta} : \boldsymbol{\eta}} - \sigma_y, \quad (5)$$

where $\sigma_y = \sigma_y(T)$ is the uniaxial yield strength,

$$\boldsymbol{\eta} = \mathbf{s} - \boldsymbol{\alpha}, \quad (6)$$

where $\boldsymbol{\alpha}$ is the back-stress tensor and $\mathbf{s}$ is the deviatoric stress, given by

$$\mathbf{s} = \boldsymbol{\sigma} - \frac{\text{tr}(\boldsymbol{\sigma})}{3}\mathbf{I}. \quad (7)$$

The evolution of the back-stress is governed by Ziegler's rule, which can be written as

$$\dot{\boldsymbol{\alpha}} = \frac{k}{\sigma_y}(\mathbf{s} - \boldsymbol{\alpha})\dot{\epsilon}_{\text{eff}}^p, \quad (8)$$

Table 1: Material properties for the frictional heat analysis.

	Disc	Pad	Plate
Thermal conductivity [W/mK]	47	0.5	46
Young's modulus [GPa]	92.9	2.2	210
Poisson's ratio [-]	0.26	0.25	0.3
Thermal expansion coefficient [$10^{-5}/\text{K}$]	1.55	1	1.15
Density [kg/m^3]	7200	1550	7800
Heat capacity [J/kgK]	507	1200	460

where $k = k(T)$ is the kinematic hardening modulus and the effective plastic strain ϵ_{eff}^p is defined by

$$\epsilon_{\text{eff}}^p = \int_0^t \sqrt{\frac{2\dot{\epsilon}^p : \dot{\epsilon}^p}{3}} dt. \quad (9)$$

The plastic strain ϵ^p is governed by the following associative law

$$\dot{\epsilon}^p = \dot{\lambda} \frac{3}{2} \frac{\boldsymbol{\eta}}{\sqrt{\frac{3}{2}\boldsymbol{\eta} : \boldsymbol{\eta}}}, \quad (10)$$

where $\dot{\lambda}$ is the plastic multiplier, which is governed by the following Karush-Kuhn-Tucker conditions:

$$\dot{\lambda} \geq 0, \quad f \leq 0, \quad \dot{\lambda}f = 0. \quad (11)$$

4 Material properties

The disc, pad and back-plate are considered in the frictional heat analysis, which is presented in Section 2. Temperature independent material properties used for these parts are listed in Table 1.

The brake discs are casted in a grey iron alloy. During stress analysis, temperature dependent material properties of the grey iron alloy are required. Most of these material parameters are obtained from measurements. Young's modulus, the yield strength and hardening behavior are obtained from tensile tests performed at 20°C, 200°C, 400°C, 600°C and 800°C. In Table 2, temperature dependent mechanical properties of the grey iron alloy are given. Young's modulus for temperatures above 800°C and Poisson's ratio is chosen in accordance to [12]. In Fig. 3 the true stress as a function of the true plastic strain for different temperatures is plotted. The solid curves show the results of the tensile tests. A curve (not shown) representing a

Table 2: Temperature dependent mechanical properties of the grey iron alloy.

Temperature	Young's modulus	Poisson's ratio
[°C]	[GPa]	[-]
20	92.99	0.26
200	83.80	0.26
400	80.45	0.26
600	67.57	0.26
800	49.84	0.26
1120	0.5	0.26
1160	0.5	0.49
2000	0.5	0.49

smoothed average of the tensile tests performed at a specific temperature is obtained by the method of least squares. The dashed line is then obtained by connecting the first and last point of this fitted curve with a straight line and is the one used as input for the temperature dependent linear kinematic hardening material model. Thus, the dashed line represents a linear

Table 3: The temperature dependent thermal expansion coefficient of the grey iron alloy.

Temperature	Thermal expansion coefficient
[°C]	[$10^{-5}/\text{K}$]
20	1.55
87	1.55
154	1.56
221	1.56
288	1.57
356	1.58
423	1.59
490	1.61
557	1.63
625	1.65
637	1.77
650	1.9
662	2.03
675	2.17
1120	2.17
1120.5	0
1200	0

approximation of the tensile tests performed at a specific temperature. The hardening behavior above 800°C is assumed in accordance to [12].

The thermal expansion coefficient of the grey iron alloy, for temperatures up to 1000°C, is calculated from experiments. In Fig. 4 the solid curve represents the results from the experiments. For temperatures above 1000°C and below solidus temperature, the thermal expansion is assumed to have the same slope as for temperatures around 1000°C. For even higher temperatures no expansion is assumed. In Table 3 the temperature dependent thermal expansion coefficient of our grey iron alloy is given.

5 Numerical results

The assembly of the disc-pad system considered in this paper is shown in Fig. 5. The disc is geometrically symmetric about a plane normal to the z-axis. It is assumed that thermo-mechanical loads applied to the system are symmetric so only a half of this assembly is considered for the simulation and symmetry constraints are applied on the nodes lying on the symmetry plane. Some detailed geometry at the inner radius that is not important for this analysis has been removed to simplify the model. The displacements along x and y directions of the nodes located at the inner radius of the disc are set to zero. All the surfaces of the disc, except the one lying on the symmetry plane are considered to lose heat by convection.

The brake pad is supported by a steel plate at the back side as shown in Fig. 6. Some detailed geometry of this back-plate which is not necessary for the simulation has been removed. Two cylindrical pins apply a normal force on the back surface of the support plate which transmits it to the pad. Displacements at the back surface of the support plate, other than along the force direction, are fixed. Furthermore, the temperature is set to zero on the back surface. The disc is meshed such that it has smaller elements where it contacts the pad as shown in Fig. 7 (only a small portion of the disc is shown). This is an advantage of the Eulerian approach because the finite element mesh of the disc does not rotate relative to the brake pad but the material flows through the mesh. In the Lagrangian approach a fine mesh should be applied on the complete surface of the disc because the finite element mesh of the disc rotates relative to the brake pad or some adaptive strategy should have to be applied. All the parts considered for the simulation are meshed with 4-noded linear tetrahedron elements by using HyperMesh (HyperWorks 10.0). These meshed parts are then used to prepare the input file with boundary conditions and loads by using Abaqus/CAE.

For the brake disc presented above, we study repeated braking by using a temperature history for three similar brake cycles. The complete temperature history is obtained by solving the thermo-mechanical contact problem for one brake cycle and then merging the corresponding temperature history

(a) 20°C

(b) 200°C

(c) 400°C

(d) 600°C

(e) 800°C

(f) 1120°C

Figure 3: The true stress as a function of the true plastic strain for different temperatures.

Figure 4: The strain as a function of the temperature during the thermal expansion coefficient test.

Figure 5: The assembly of the disc-pad system.

Figure 6: The brake pad with the support plate (back-plate).

Figure 7: The mesh of the disc showing the fine mesh density at the contact zone.

Figure 8: After the brake application, a ring of high temperatures develops on the disc surface.

three times in a sequence by using the script discussed in Section 2.

In Fig. 8, temperatures of the disc surface are shown for the frictional heat simulation of one brake cycle. A brake force of 24.5 [kN] is applied for 20 [s] on the back surface of the support plate. The angular velocity of the disc is 45 [rad/s] and held constant throughout the simulation. The force is ramped up by using a log-sigmoid function during 20 time increments and then held constant for the next 80 increments with time step $\Delta t=0.2$ [s]. This loadcase corresponds to a truck moving downhill with a constant speed. The friction coefficient is $\mu = 0.3$, the contact conductance is $\varphi = 0.1$ [W/NK] and the convection coefficient is set to 50 [W/m²K]. After braking, the disc is cooled for another 5000 [s] with a time step of 5 [s] in order to bring it back to the ambient temperature. The disc assembly is meshed with 270194 elements.

In Fig. 9, a graph of the temperature at a node on the disc is shown. It

Figure 9: The temperature history for a node on the disc surface.

can be seen that the same temperature history is repeated three times for the repeated braking simulation. The node chosen for this plot is located on the disc surface at 180° from the middle of the pad and at a radius of 163.9 [mm]. Here it is important to mention that the outer radius of the disc is 218.3 [mm] and inner radius is 112.3 [mm]. The total time, i.e. sum of the braking and the cooling time for one brake cycle has been scaled so that it appears as a unit on the time scale.

The temperature history of this frictional heat analysis is imported to Abaqus and a stress analysis is performed. The total CPU time for the frictional heat and stress analysis is 3.05 [h] and 5.54 [h] respectively, on a workstation with Intel Xeon X5672 3.20 GHz processor. For comparison purpose, a simulation was performed for one brake cycle in a commercial software on the same workstation using a total Lagrangian framework. The brake force was applied only for 0.1 [s]. Otherwise its magnitude and the boundary conditions were the same as used in our sequential approach. After braking the disc was not cooled down. The total CPU time for the analysis is 40.12 [h]. This shows that a tremendous amount of computational time is required for the total Lagrangian approach even though the analysis was run for a braking period of only 0.1 [s]. In order to simulate the braking of 20 [s] studied in this work, approximately 8024 [h] will be needed if a total Lagrangian approach is applied.

Below some results from the stress analysis when using the sequential

Figure 10: The circumferential stress at a node on the disc surface.

approach are described at the same node for which the temperature history was shown in Fig. 9. In Fig. 10, a graph of circumferential stresses on the disc surface is shown. It can be seen that during each braking cycle as the brake force is applied, circumferential compressive stresses are generated. But after the disc cools down, they transform into tensile stresses. By comparing it with figure 9, it can be seen that there is a decrease in the magnitude of the compressive stresses even when the nodal temperature is rising. This reduction in stresses can be partly explained by the reduction in hardening of material at high temperatures. In Fig. 11, a graph of the von Mises stresses on the disc surface is shown which elaborates that the stresses go beyond the yield limit of the material. A graph of the corresponding circumferential plastic strains of the disc material at the surface is shown in Fig. 12. By studying these two figures, one can see that the material is deformed permanently first in compression when the brake force is applied and then in tension when the disc cools down. From Fig. 12 it can be seen that for the first cycle the yielding in tension is less than the yielding in compression so there is a residual compressive strain after the disc cools down. But during subsequent cycles the yielding in tension is equal to the yielding in compression. So the magnitude of residual compressive strain remains same after each cycle. Similarly, von Mises stress graph shows a residual stress after the first cycle and at the end of each subsequent cycle its magnitude remains unchanged. This behavior is attributed to the kinematic hardening model

Figure 11: The von Mises stress at a node on the disc surface.

and that the same load history is used during each cycle.

The compressive stresses on the disc surface can be explained by the expansion of the surface material due to the increase in temperatures. The frictional heat causes the disc surface temperature to rise in a short period of time as compared to the inner region of the disc as shown in Fig. 8. Consequently, the surface expands more than the inner region of the disc and due to the closed shape of the disc, compressive stresses are generated on the surface. When these stresses surpass the yield limit, the material deforms permanently. But when cooling down the disc, these compressive stresses transform into tensile stresses because the material has already yielded in compression. Depending upon the magnitude, the residual tensile stresses in circumferential direction may cause the initiation of radial microcracks on the disc surface.

In Fig. 13, a graph of radial stresses on the disc surface is shown. It can be seen that as the brake force is applied, initially compressive stresses are generated. By comparing it with figure 9, it can be seen that the magnitude of compressive stresses start decreasing significantly even when the nodal temperature is rising. This reduction of stresses can be partly explained by the higher expansion of the surface material in radial direction as compared to the inner of the disc in the beginning but as the heat flows to the inner region, it also expands radially. So due to the reduction in difference in the expansions, the compressive stresses start reducing in mag-

Figure 12: The circumferential plastic strain at a node on the disc surface.

nitude already during braking. The behavior of the disc is different in radial direction from circumferential direction mainly because in circumferential direction expansion is constrained due to round shape but in radial direction it is relatively free to expand. After the disc cools down it can be seen that stresses transform to residual compressive stresses. By comparing with Fig. 10, it can be concluded that the radial stress magnitudes are quite lower in compression but are almost the same in tension. Furthermore, the residual radial stresses are compressive, but the residual circumferential stresses are tensile. This is indeed in agreement with the observation that radial micro-cracks on disc surfaces are more marked than circumferential ones, even when macroscopic cracks do not appear [1]. In Fig. 14, a graph of circumferential stresses against different measures of the strain in circumferential direction for the repeated braking is plotted. It can be seen that after the first cycle, the behavior becomes approximately stable. It can also be seen that the thermal strain has higher range than the mechanical strain ($\epsilon^e + \epsilon^p$), so it influences the graph of the total strain more. The shape of the graphs can be explained by the temperature dependent thermal expansion coefficient and the von Mises material model. Such stress-strain cycles might generate radial cracks after a few braking operations resulting in low cycle fatigue of a disc brake. Now if the fatigue life data for the disc material is known, its fatigue life can be assessed. It is clear that material on the disc surface undergoes substantial plastic strain, which suggests that a strain-based

Figure 13: The radial stress at a node on the disc surface.

Figure 14: Four different stress-strain graphs for three repeated braking operations.

approach is needed for predicting the fatigue life.

6 Discussion

During the frictional heat analysis, temperature independent material data has been used. For more realistic results, temperature dependent material data should be used. The friction coefficient of a brake pad is generally dependent on temperature, velocity and contact pressure [13] but in this work it is assumed to be constant at $\mu = 0.3$ to represent an average behavior. In a very near future, we will extend this work such that a temperature dependent behavior of the friction coefficient is included in the proposed method. At present the in-house software assumes constant angular velocity of the disc but in the near future it will also include non-constant angular velocities. In the stress analysis, a material model has been used that assumes the same behavior of the material both in tension and compression in monotonic loading but in reality cast iron has different properties in tension and compression. So in the future it would be advantageous to implement a material model incorporating the different behavior of cast iron in tension and compression. Such a material model has been developed by Koetniyom et al. [5]. It would also be more realistic to use nonlinear kinematic hardening model for studying repeated braking. Furthermore, in the future a proper low cycle fatigue criteria will be implemented in our proposed sequential approach.

7 Conclusions

In this work a sequential approach has been implemented to study thermal stresses in disc brakes for repeated braking. Frictional heat analysis is performed in an in-house software based on an Eulerian approach and stress analysis is performed in a commercial software, Abaqus, which uses the thermal history from the frictional heat analysis as input. This method has proved tremendously cheap in terms of computational time when compared to a fully coupled Lagrangian approach.

The stress analysis results show that during hard braking, high compressive stresses are generated in the circumferential direction on the disc surface which cause plastic yielding. But when the disc cools down, these compressive stresses transform to tensile stresses. For repeated braking when this kind of stress-strain history is repeated, stress cycles with high amplitudes are developed which might generate low cycle fatigue cracks after a few braking cycles.

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References

- 1 **Dufrénoy, P. and Weichert, D.** A thermomechanical model for the analysis of disc brake fracture mechanisms. *Journal of Thermal Stresses*, 2003. **26**(8), pp. 815–828.
- 2 **Gao, C., Huang, J., Lin, X. et al.** Stress analysis of thermal fatigue fracture of brake disks based on thermomechanical coupling. *Journal of Tribology*, 2007. **129**(3), pp. 536–543.
- 3 **Richmond, J., Kao, T. and Moore, M.** The development of computational analysis techniques for disc brake pad design. *Proceedings, Advances in Automotive Braking Technology: Design Analysis and Material Developments*, 1996.
- 4 **Kao, T., Richmond, J. and Douarre, A.** Brake disc hot spotting and thermal judder: an experimental and finite element study. *International Journal of Vehicle Design*, 2000. **23**(3), pp. 276–296.
- 5 **Koetnityom, S., Brooks, P. and Barton, D.** The development of a material model for cast iron that can be used for brake system analysis. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 2002. **216**(5), pp. 349–362.
- 6 **Choi, J. and Lee, I.** Finite element analysis of transient thermoelastic behaviors in disk brakes. *Wear*, 2004. **257**(1-2), pp. 47–58.
- 7 **Kim, D. J., Seok, C.-S., Koo, J.-M. et al.** Fatigue life assessment for brake disc of railway vehicle. *Fatigue & Fracture of Engineering Materials & Structures*, 2010. **33**(1), pp. 37–42.
- 8 **Šamec, B., Potrč, I. and Šraml M.** Low cycle fatigue of nodular cast iron used for railway brake discs. *Engineering Failure Analysis*, 2011. **18**(6), pp. 1424–1434.
- 9 **Nguyen-Tajan, T., Thomas, J., Houari, M. et al.** A Computationally Efficient Eulerian Algorithm for Sequentially Coupled Thermal Mechanical Analysis of a Solid Brake Disc. *ABAQUS Users' Conference, Stockholm, Sweden*, 2005, pp. 363 – 372.
- 10 **Strömberg, N.** Development and implementation of an Eulerian approach for efficient simulation of frictional heating in sliding contacts.

In *IV International Conference on Computational methods for Coupled Problems in Science and Engineering*. Eccomas, 20-22 June, Kos, Greece, 2011.

- 11 **Strömberg, N.** An Eulerian approach for simulating frictional heating in disc-pad systems. *European Journal of Mechanics - A/Solids*, 2011. **30**(5), pp. 673 – 683.
- 12 **Gustafsson, E., Hofwing, M. and Strömberg, N.** Residual stresses in a stress lattice—Experiments and finite element simulations. *Journal of Materials Processing Technology*, 2009. **209**(9), pp. 4320 – 4328.
- 13 **Heussaff, A., Dubar, L., Tison, T. et al.** A methodology for the modelling of the variability of brake lining surfaces. *Wear*, 2012. **289**, pp. 145–159.

Appendix

Notation

α	Thermal dilatation coefficient
$\boldsymbol{\alpha}$	Back-stress tensor
$\boldsymbol{\epsilon}$	Infinitesimal strain
$\boldsymbol{\epsilon}^e$	Elastic strain
$\boldsymbol{\epsilon}^p$	Plastic strain
$\boldsymbol{\epsilon}^t$	Thermal strain
$\boldsymbol{\sigma}$	Cauchy’s stress
$\boldsymbol{D}$	Elasticity tensor
$\boldsymbol{I}$	Identity matrix
$\boldsymbol{s}$	Deviatoric stress
$\dot{\lambda}$	Plastic multiplier
ϵ_{eff}^p	Effective plastic strain
σ_y	Uniaxial yield strength
k	Kinematic hardening modulus
T	Temperature

T_{ref} Reference temperature

T_i Initial temperature