Yield Curve and its Predictive Power for Economic Activity
The Case of USA

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Abstract

This paper attempts to examine the statistical relationship between the treasury yield spread and the possible predictive significance it may carry for real economic activity in the United States. We conduct a study of the theories of the yield curve along with an investigation into the variables that give rise to its predictive power. The chief emphasis lies upon an empirical study conducted using the non-linear probability model, the Probit approach. Our results indicate that the yield spread is indeed a strong macroeconomic variable for predicting the probability of a recession with a lead of four quarters as the model effectively forecasts the seven previous recessions in the U.S. economy.
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1 Introduction

Predicting the future has always fascinated human minds and in particular, economic forecasting, which doubles the incentives due to its potential for exploiting profits. One widely popular forecasting tool is the yield curve, which has come into the limelight in the past two decades due to the steady increase in the size of government budget deficits in most of the developed world. This has led to a dramatic rise in government borrowings primarily via means of issuing bond securities of differing terms¹ (Supply side factor). Another reason for its growing popularity is the increased interest of investors in sovereign bond securities following the dot com crisis and the recent financial meltdown, which has made investors increasingly fearful of investments in equities² (Demand side factor). Consequently, a profound understanding of the yield curve and its possible implications for economic growth has become an important area of research.

The former Fed Chairman, Alan Greenspan, in 2005, writes in a letter to Jim Saxon:

“The key component from which the yield curve slope derives much of its predictive power for future GDP growth” is “the gap between the current and long-run levels of the real federal rate”.


This statement aptly signifies the relevance of yield spread for future economic growth.

Given that we cannot know we are in a recession until we are already in one for a while, an investigative study into predicting the likelihood of a recession ahead of time becomes all the more important. This is of grave relevance to businesses so they can plan their cash flows ahead of time, and to policy makers so they can design a policy framework should an economic downturn occur.

The Treasury yield curve which shows the prevailing yield to maturity (YTM) of various US government bond maturities has historically outperformed the Commerce department’s index of leading indicators, S&P 500 returns and the Stock and Watson XLI index in forecasting economic activity with a lead time of four quarters and beyond. The commerce department’s index of leading indicators comprises of fluctuations in 11 individual variables, which are combined together in a weighted average. The Stock and Watson XLI uses a complicated modeling framework that takes into account seven individual series, each with several lags. The relative superior ability of the yield curve to predict future economic activity compared to the mentioned indicators, is supported by statistical evidence provided by Dueker (1997), Dotsey (1998), and Estrella and Mishkin (1998).

One may wonder why we should resort to financial indicators when there is a whole array of macroeconomic models available that help predict future levels of economic activity. We argue that in case those models produce results that are in agreement with

¹ See Appendix A for charts illustrating US budget debt and deficit as a percentage of GDP. (Figure A 1 and Figure A 2)

² See Appendix A for a chart showing Bond Fund Inflows against Equity Fund Inflows. (Figure A 3)
what the financial indicators tell, confidence may be enhanced in the model. However, if the results are different, the model may not necessarily be wrong but it may be worthwhile to review the assumptions underlying it. Moreover, financial indicators are usually readily available, simple, and quick to interpret. Of course, this assumes that the indicators are accurate and have relevance for forecasting. In short, financial indicators can supplement and crosscheck the forecasts produced by sophisticated macroeconomic models.

The Treasury yield curve has correctly predicted the past seven US recessions including the recent financial crisis with at least a lead of four quarters. At all of these occasions, the spread between the long and short treasury instruments has narrowed and become negative\(^3\). Therefore, an inversion of the curve may be interpreted as an indicator of worsening economic conditions ahead. However, some economists point out to the two “near misses” in 1995 and 1998 when a narrowing of the treasury yield spread did not follow an economic downturn ahead.

In our paper, we use the case of USA as the treasury market is largely considered default free due to the facts that US government has never defaulted on its obligations and the historic strong outlook of the US economy. However, other risk factors such as inflation and currency risk to foreign investors exist. Additionally, the US bond market is considered the most liquid bond market in the world. A liquid bond market is characterized by the participants’ ability to buy and sell bond securities at the equilibrium price at all times. Moreover, the Treasury bond yields exert considerable influence over the well-being of the global financial system. It is the benchmark yield indicator of the rest of the global debt capital market, and the spread over the Treasury yield is used to assess the creditworthiness of any issuer. The Fed holds an auction for issuance of treasury securities. The following three types of securities are issued:

- Treasury bills – zero coupon bonds of maturities between one month and one year.
- Treasury notes – semi-annual coupon bonds of maturities between one and ten years.
- Treasury bonds – semi-annual coupon bonds of maturities between ten and thirty years.

In addition to its ability to make predictions about future economic activity, the Treasury yield curve has numerous other uses including:

- The yield curve is of central importance to monetary policy makers who consider it an important criterion when assessing current policy framework or devising future policies.
- The yields on the sovereign bonds help in the pricing of all other debt capital instruments. For example, if the yield on a 5-year T-Note were 4% then all other 5-year bonds, whosoever issues them, would be issued at a yield of above 4%.
- Fixed income portfolio managers heavily delve into the yield curve to maintain a delta neutral portfolio. Additionally, they may analyze the yield curve to assess

\(^3\) See Appendix A for a chart displaying US treasury yield spread and recession dates. (Figure A 4)
which segment of the curve offers the maximum return in comparison to all other segments.

- The shape of the yield curve can help in identifying bonds that are likely to be overpriced or underpriced. For example, a longer-run maturity bond that offers a lower yield compared to a short-run bond, could be an indicator that it was trading dear to the curve.
- The yield curve plays a crucial role in the pricing of a vast majority of interest linked derivative instruments such as swaps, futures and other hybrid instruments.

1.1 Purpose

This paper aims to identify if there exists any statistical relationship between the treasury yield spreads and economic activity in the US via a Probit approach and to quantify the predictive power of the spread in probabilistic terms. The objective is to determine whether this relationship can be used to forecast future economic outlook and for that reason, we intend to find the optimum threshold for the occurrence of a recession.

1.2 Disposition

The paper is divided into four sections. First, we provide a selective literature review to study the previous research conducted on the topic in hand. The second section is concerned with a theoretical study of the underlying relationship and various proposed theories explaining the possible shapes of the yield curve. We then move to structuring a methodology, which entails data collection and various details regarding our choice of regression model. Finally, we investigate an empirical study on the given data set and conclude our findings.
2 Selective Literature Review

Examination of interest rates’ behavioral aspects of different maturities throughout the course of the business cycle goes back notably to Mitchell (1913). Nonetheless, Kessel (1965) was the pioneer to bring about a peculiar, clearly outlined reference to the behavior of interest rates. He revealed that many term structural spreads are more likely to be depressed at the kickoff of recessions – the peak of the business cycle – and high as economic expansions come forth subsequent to cyclical troughs. Butler (1978) associated the yield curve as a forecaster of future short-term interest rates, to the inferences of the diminishing of those rates for contemporaneous economic activity, suggesting some of the later reasoning. He unmistakably forecasted the absence of a recession in year 1979. Towards the end of the 1980s, an outburst in research on the yield curve as a prominent indicator arose.

As the literature on the importance of the interest rate differential, or what is more commonly known as the yield spread, in predicting economic growth, is quite extensive; not all papers can be considered in this section. Publications concerning the yield spread-economic growth relationship dates back to the 1980s; Harvey (1988, 1989), Stock and Watson (1989), Chen (1991), Estrella and Hardouvelis (1991) and many others, pointed out that a negative interest rate differential might be a precursor for a forthcoming recession. Those studies aimed at predicting future growth in the gross domestic product using U.S. financial data and found exceptionally significant dependence between real economic activity and the yield spread with lead horizons of one to eight quarters ahead.

Many investigators provide evidence that shows that the term structure of interest rates has predictive capacity and is adequate for forecasting purposes. As it appears, it is also compatible with predicting real economic activity. Kessel (1965) brings forth this empirical invariability. Later, Fama (1986) further elaborates on the topic without supplying any statistical attestation or manifestation. Laurent (1988, 1989) used the lags of the interest rate differential between long-term and short-term rates to predict the growth in real GNP. Despite the fact that the summation of the lagged interest rate differentials was positive, the results were insignificant. Harvey (1988) studied the relationship between the real interest rate differential and the succeeding real consumption growth via the Consumption Capital Asset Pricing Model (CCAPM). He brought forth a number of valid practical corroborations. Campbell and Shiller (1987) recognized further evidence in line with the hypothesis that suggests that the term structure contains important data regarding the future progress and movement of interest rates. Furlong (1989) conveyed his apprehension and uncertainty regarding the reliability of the yield curve as a leading and prominent indicator. Nonetheless, he moderately marked its predicting power for economic downturns and recessions.

In empirical papers, the spread has been computed as the arithmetic difference between long-term interest rates and short-term interest rates. Some papers have veered from this prototype via the usage of, for instance, the Fed rate as their respective short-term interest rate, or by experimenting with different forecasting horizons. Two different classes of the regressand have been put to the test: continuous variables (e.g. GDP or GNP growth in real terms) and discrete variables (e.g. binary dummy recession variable).
Estrella and Hardouvelis (1991) used an empirical approach via the introduction of a probit model to show that the yield curve may be employed to forecast real growth in investment, consumption, aggregate gross national product (GNP) and the recessions dated by the NBER (National Bureau of Economic Research). The model did very well and was prominently reliable using with U.S. statistics. They explicitly examined the optimal horizon and discovered that four to six quarters ahead yield the most significant results. Bomhoff (1994), Davis and Henry (1994), and Davis and Fagan (1997) have conducted similar research and realized invariable and harmonious results.

Bernard and Gerlach (1998) and Estrella and Mishkin (1998) extended and built on the previous analysis. The first widened the compass to eight countries and did not limit the analysis to the case of the U.S. only. The latter investigated into a spectrum of forecasting horizons and examined the out-of-sample forecasting performance in contrast to alternative financial variables and found out that the yield spread and stock price indices are the most significant financial indicators as they are capable of forecasting recessions with a lead horizon of four quarters.

Other papers looked into the predictive power of the yield spread in different countries and regions than the United States. Both, Plosser and Rouwenhorst (1994) and Davis and Henry (1994) investigated the case for Germany and the UK while Davis and Fagan (1997) looked into the case of the EU. Those three papers, in addition to Estrella and Mishkin (1997), demonstrate that the yield curve is capable of predicting real economic activity in many countries, with exceptionally significant results in Germany.

Bordo and Haubrich (2004) and Baltzer and Kling (2005) support statistical proof via a regression-based approach to reason out that the forecasting ability is not time invariant but rather varies over time and it appears to be interdependent on the verisimilitude and plausibility of monetary policy. Contra to most research papers that rely on statistical data as of the 1950s, Bordo and Haubrich (2004) based their study on U.S. data 1875-1997 while Baltzer and Kling (2005) based their study on German data 1870-2003.

Whilst only a bivariate specification is required for the incomplex model, few papers assign supplementary variables when using the model to forecast recessions. For instance, Wright (2006) reasons that the inclusion of the short-term interest rate to the Probit model when forecasting recession further enhances the results of in-sample forecasting.
3 Theoretical Framework

While there is plenty of empirical literature on the predictive power of the yield curve, economic pundits lack a well-defined and accepted theory on why the yield curve predicts future economic growth. The focal point of yield curve research has been greatly inclined towards formulating empirical models rather than developing theoretical foundations of the underlying relationship between yield spread and future economic activity. This unavailability of a universally accepted theoretical model for the relationship has made some observers skeptical of whether the yield curve can function practically as a leading indicator. If economists cannot reach a consensus on why the relationship exists, confidence in yield curve as a leading indicator may deteriorate. Nonetheless, the literature does not hesitate to characterize the spread’s predictive ability as ‘impressive’, ‘robust’ and ‘particularly well established’.

However, there have been attempts to theoretically model this relationship. Estrella and Hardouvelis (1991) and Berk (1998) refer to simple dynamic IS-LM models. Harvey (1988) uses a consumption capital asset pricing model (CCAPM). Chen (1991) and Plosser and Rouwenhorst (1994) refer to real business cycle (RBC) models. In addition, Rendu and Stolin (2003) make use of a dynamic equilibrium asset-pricing model to explain the relationship between the term structure and real activity. However, a theoretical study of these models is beyond the scope of this paper.

We begin our theoretical study by first dissecting the phenomenon of the ‘yield to maturity’, which is central to the theme of our paper. We then turn our attention to studying the various shapes of the treasury yield curve observed in real world and relate them to a set of economic theories. Finally, we look into some of the most important factors that give rise to the predictive ability of the yield curve.

3.1 Yield to Maturity (YTM)

The yield to maturity on a bond accounts for both the coupon payments from the bond and the price paid for it. It is the discount rate, which equates the future streams of cash flows offered by a bond security to its current market price. Hence, it is the compensation that an investor earns by receiving the present value of capital gains, coupon payments and the par value in comparison the market price of the bond.

The methodology involving the computation of the yield to maturity follows an iterative numerical approach using the bond price formula and solves for the interest rate according to the following formula:

\[
\text{Bond Price} = \text{Cash Flow} \times \frac{1}{\left(\frac{1}{1+\text{interest rate}}\right)^n} + \left[ \text{Maturity Value} \times \frac{1}{(1+\text{interest rate})^n} \right] \quad (\text{Eq. I})
\]

4 These are quoted, respectively, from Estrella and Hardouvelis (1991, p. 559), Kosicki (1997, p. 39), and Hamilton and Kim (2000, p. 3).
To grasp an understanding of it, we first observe the relationship between a bond’s price and its yield. In general, as a bond's price increases, its yield decreases. The relationship is represented in Figure 1 below.

This is due to the fact that rising market interest rates (which go hand in hand with bond yields) would make the fixed coupon rate on a bond security less valuable to other investors; as a result, the price the investors would be willing to pay for this security falls. For example, assuming the current market rates are 5% and the bond is trading at par: if the rates go up to 6% next month but the bond security still offers coupon payments at a rate of 5%, naturally, the price of the respective bond security is expected to fall in the next month. This brings us to the conclusion that if the coupon rate is higher than the market interest rate then the bond is trading at premium, and if the coupon rate is lower than the market interest rate then the bond is trading at discount.

While the yield to maturity is the most common measure of a bond return in financial press, it has its limitations. First, it assumes that the bondholder would hold the bond security until maturity. Second, it assumes that every coupon payment received is reinvested at the same YTM. This is unrealistic as the market rates change over time. However, it remains the most appropriate measure as it accounts for both the time value of money and capital gains between the purchase date and maturity date.

3.2 Types of the Yield Curve

The treasury yield curve is simply a diagrammatic illustration depicting the yield to maturity, on the y-axis, against time to maturity, on the x-axis. It serves in assessing the relationship between bonds of the same issuer that carry similar credit risk. We can identify four main types of the Yield Curves:

**Normal Yield Curve**

The normal yield curve is the most commonly observed shape of the term structure of interest rates. It displays an upward slope that depicts the variation of the yields or interest rates as a function of time or maturity. The positive slope of the normal yield curve indicates that as time to maturity lengthens, the yields increase. Generally, this shape of
the yield curve reflects the market’s expectations for the economy to grow. The mean spread of the yield curve for the period 1976-2006 was 83 basis points implying that the yield curve on average has been a normal one.

**Inverted Yield Curve**

The inverted yield curve – also known as the negative yield curve – has always preceded recessions and economic downturns. The downward slope of the yield curve shows that as time to maturity increases, the yields decrease, rather than increase. This shape of the yield curve depicts negative sentiments in the market as it shows that longer-term yields are lower than shorter-term yields. This indicates that the market is anticipating a decrease in the risk free rate. Nonetheless, this scenario seems to be a paradox as it contradicts logic! In the time where short-term investors’ investments are yielding high interest rates, why would long-term investors settle for a lower yield? This is justified by the intuition that investors expect interest rates to go further down and decline even more in the future and that is why they accept the current low interest rates as a way to lock this low rate before the economy deteriorates further.

**Flat or Humped Yield Curve**

As the yield spread differential between short-term and long-term interest rates diminishes, the yield curve begins to flatten. The flat yield curve means that regardless of the duration of holding a fixed income asset, the yield is the same.

The humped yield curve – also known as the bell shaped yield curve – shows that short-term yields and long-term yields are equal while somewhere in the middle run, yields tend to be higher.
Both, the flat yield curve and the humped yield curve, reflect uncertainty in the economy and confusion in the market towards the future of the term structure of interest rates. This curve could later slope positively and become a normal yield curve or negatively and become an inverted yield curve. Thus, we can say that a flat or humped yield curve is a transition curve.

3.3 Theories of the Yield Curve

The following theories attempt to explain the variations in the shape of the treasury yield curve that have been described above:

Expectation Theory

Irving Fisher (1896) originally theorized this theory in his paper *Appreciation and Interest Rate*. However, Hicks (1939) and Roll (1970) undertook modern development of the theory.

There are two main competing variations of the expectation theory:

1. *Local expectation hypothesis*: The theory implies that bonds that share similar class but vary in terms of maturities should have the same expected holding period rate of return. For example, an investor who intends to hold a bond for six months should get the same return regardless of which specific bond he buys.

   Theoretically, it may appear very convincing since in a no-arbitrage world, any discrepancy between the holding period returns of different maturity bond instruments would be immediately cashed out by arbitrageurs. However, this is not very consistent with what we observe in practice where the holding period returns from longer-dated bonds are higher than those from short-dated bonds. The higher holding period returns on longer run bonds are coherent with the risk-reward theory, which proposes higher returns for higher risk taking. Longer-run bonds are more vulnerable to interest rate changes and carry greater risk; therefore, they require a positive spread over the holding period returns of short-run bond instruments. Additionally, the discrepancy in the holding period return could be attributed to the restrictions imposed onto investors regarding the bonds they can hold. For example,
banks are required to hold short-run bond instruments to maintain liquidity. Therefore, even in an environment of economic disequilibrium where banks could potentially gain from holding long-term bonds, they may still have to hold on to short-dated bonds.

2. **Unbiased expectation hypothesis:** This is a particularly useful theory in explaining the shape of the yield curve. The theory holds that future spot interest rates are unbiased estimators of current implied forward rates. Therefore, according to the unbiased expectation hypothesis, a positively sloped yield curve corresponds to the belief that market expects future spot interest rates to rise. Similarly, an inverted yield curve corresponds to the belief that the market expects future spot interest rates to fall. If the market expects the future short-run rates to increase, then the long run rates should be higher than the current short run rates to reflect this. Conversely, if the market expects the future short-run rates to fall then the long run rates should be lower than the current short-run rates. In short, the long run rate is the geometric average of current and expected future short-term rates. Therefore, a long-term investor should expect to earn the same amount of return via successive investments in short term bond instruments that he would normally earn by investing in a long-term bond instrument.

The expectations are largely a function of the expected rate of inflation. Usually, in the presence of inflationary pressure, the yield curve would be upward sloping and in the absence of inflationary pressure, the yield curve would be downward sloping. One major criticism of the unbiased expectation theory is that it makes a rather unrealistic assumption concerning an investor’s ability to make accurate forecasts of very long-dated spot rates. For example, it assumes that an investor would be able to foresee thirty years into the future for a thirty-year T-bond yield. Nevertheless, the information is embedded in the yield of a thirty-year bond and expresses the market view on inflation and future interest rates for thirty years forward.

**Liquidity Premium Theory**

This theory was put forward by Hicks (1939). It proposes that whilst expectations play a crucial role in determining the shape of the yield curve, investors have a natural inclination for short-term issues because they are considered more liquid. Securities that mature over a longer period are inherently more risky than those that mature over a shorter period. For example, it is intuitive for a lender to demand a higher rate of return from long-term lending than from short-term lending, as there is a greater chance of the borrower defaulting over the course of longer period than over that of a brief one.

Based on our understanding of the expectation hypothesis, we would expect a flat yield curve in an environment of stable inflation such as when expected future short run rates are the same as current short run rates. However, in practice, we observe a positively sloping yield curve even in the absence of inflationary pressure and this phenomenon may be explained with reference to the liquidity preference theory. Generally,
borrowers wish to borrow over long-term and lenders wish to lend over short-run. To address this mismatch, lenders are induced to lend through compensation in form of liquidity premia. In other words, it is the compensation required for investing in a relatively illiquid security. This risk premium is above the average of current and expected future short run rates and it increases further investors’ lending across the yield curve. To give an account of the relevance of the theory to a negatively sloping yield curve, we prognosticate that the expected fall in future short run rates is expected to offset the increase in rates brought by the liquidity premium. In the absence of liquidity risk premium, investors would hold a bias for short-run securities in an attempt to minimize the variability in the value of their bond portfolios.

**Segmentation Theory/ Preferred Habitat Theory**

Culbertson (1957) and Modigliani and Sutch (1966) argue that the liquidity premium is not the only additional consideration that investor take into account when making decisions on the horizon of their investments. The theory proposes that the short run and long run market for bond securities is effectively segmented in such a way that there is no inter-relationship between them. The short run yield is determined through the interaction of short run borrowers and lenders and the long run yield is determined through the interaction of long run borrowers and lenders. Moorad Choudhry argues, “The shape of the yield curve is determined by the demand and supply of certain specific-maturity investments, each of which has no reference to any other part of the curve” (Choudhry, 2006, p. 123).

In general, we observe a lack of demand for medium-term securities on investors’ behalf. This may explain for the humped shaped yield curve. Therefore, if there were no demand for one specific maturity investment, then we would expect its yield to lie above other segments. Furthermore, the investors do not regard bonds of varying maturities as perfect substitutes. For example, banking institutions have an inherent bias for short-term instruments due to high liquidity requirements whereas pension funds usually tend to invest in longer run instruments.

A modified version of this theory is the preferred habitat theory, which claims that whilst market participants have interest in specific segments along the yield curve, they may be tempted to invest in other parts of the maturity spectrum if they see a good value. For example, banks may at times be tempted to invest in long-dated securities if the yield is high enough to compensate for the associated risk. Essentially, the theory holds that investors show a reasonable degree of flexibility about their investing decisions.

**3.4 Limitations of the Theories of the Yield Curve**

Based on our theoretical analysis, we can conclude that the expectation hypothesis combined with liquidity premium theory does rather well in explaining the shape of the normal, inverted, and flat yield curves while the segmentation theory is particularly useful in explaining the humped shape of the yield curve. However, these theories are subject to certain assumptions of a frictionless capital market, which has the following features:
• Perfect information and rationality: All market participants are not only assumed to have access to all relevant information but also rationally account for all the information in their decision-making. In practice, not all information is freely available, and market participants do not always behave rationally, particularly in periods of crises.

• Bullet maturity bonds: This assumes that the only class of bond instruments issued is that which cannot be redeemed before the maturity. However, in practice, there are callable sovereign bonds that can be redeemed at the will of the issuer.

• No transaction cost: It is assumed that there are neither taxes nor commissions for executing transactions. In practice, most governments charge taxes on capital gains and most brokerage firms charge a certain commission for placing market positions.

Discounting for the above factors makes the discussion about the term structure easier to comprehend. However, one should be aware that in practice, markets are far from perfect and the conventional theories may not always be able to explain for the movements in the yield curve.

3.5 Fed’s Monetary Policy & Treasury Yield Curve

Fed Policy Mechanism

The predictive power of the yield curve is derived from a variety of factors; however, we focus on how monetary policy, often quoted to be the single most important factor behind the predictive power of yield curve, affects its behavior. The Fed has various monetary policy instruments including its open market operations, discount rate, and the required-reserve ratio at its disposal to influence both short and long run rates and consequently affects the slope of the yield curve. Typically, the Fed engages in open market operations conducted on Treasury bills to target a certain federal fund rate. When the Fed conducts open market purchases of T-bills, it reduces the quantity that is available in the market and consequently, the price of T-bills increases (or the yield falls). For instance, assume the Fed intends to set the federal fund rate within the range of 5.9 – 6 percent. Its trading desk at the Federal Reserve Bank of New York stands ready to purchase any amount of bonds yielding an interest rate above 6 percent and to sell any amount of bonds yielding an interest rate below 5.9 percent. In this manner, the Fed has direct control over the short end of the yield curve.

When the Fed wishes to pursue a monetary policy ease, it lowers the federal fund rate and vice versa. The rationale is to use the rate increases to contain economic growth and curb inflation when the economy shows signs of overheating. Conversely, the Fed decreases the rate to promote economic growth by stimulating credit activity when the economy shows signs of a slow-down. For example, according to the Taylor (1993) rule the Fed should decrease the nominal yield by 1 percent for every 2 percent drop in GDP growth.

Unlike the short-term rate, the Fed has little influence over the long-term rate. It would like to influence the long-term rate through bringing adjustments to the short-term rate because it is the long-term rates that are believed to have relevance for economic
activity. Higher long-term bond yields will retard investment activities, as many of the projects that previously had positive net present values (NPV) may now need to be abandoned.

The short-term yields generally fluctuate with near-term expectations of policy, which takes account into cyclical considerations. A policy squeeze is usually required in the beginning of business cycle recoveries to offset the accommodation earlier conducted to pull the economy from recession. Towards the late stages of business cycle expansions, a policy squeeze is usually required to quench excess demand and inflationary pressure. Following this line of argument, short-term rates are expected to be highly influenced by investor’s perception of how monetary policy will be conducted to balance out variations in the business cycle. On the other hand, long-term rates tend to be associated with longer-term expectations of policy. They are highly influenced by investor’s perception of structural and fundamental features of the economy. This is because over the longer course of time, investors are likely to be of the opinion that short-term cyclical disturbances would offset each other leaving a zero net effect and returning the federal fund rate to its neutral level.

If the monetary policy comes as a surprise and upsets the expectations of the credit market, it may fail to have its desirable impact. If the Fed pursues a monetary squeeze in an environment of low inflation, the market would regard this move as over anti-inflationary and bond yields may thus fall. Conversely, if the Fed pursues monetary easing in an environment of high inflation, it could well set off an increase in bond yields as bond holders sell off their holdings as a reaction against a flawed Fed policy. The way credit markets react to a Fed’s squeeze or ease depends largely on the primary market factors that lead bond yields. The point to be noted here is that the most important of these factors are inflation expectations and the potential of investment alternatives.

However, at occasions, the Fed may be tempted to shy away from pursuing a monetary tightening even if the economic fundamentals mandate its execution. This could be the case when borrowers are highly vulnerable to changes in the short end of the yield curve, banks are not appropriately capitalized, or if monetary tightening carries certain repercussions for the United States’ main trading collaborates. Nonetheless, there is a strong bias to adjust the policy to address domestic issues.

**Relationship between Yield Curve and Subsequent Economic Activity**

Why is there a relationship between the slope of the yield curve and real economic activity? The predictive ability of the yield curve is derived from the notion that changes in curve’s slope are associated with changes in monetary policy. However, monetary policy may also respond to output, so that the yield curve reports a complex blend of policy actions. In the light of this logic, the yield curve predicts future economic activity by predicting the state of monetary policy.

There are two competing hypothesis on how monetary policy changes manifest themselves in yield curves. The first hypothesis claims that the causal variable behind slope of the yield curve and its predictive power for real economy is current monetary policy. Assume that the Fed exercises a contractionary monetary policy to cool down an
overheating economy. Consequently, the short-term rates spike. It is expected that a policy ease would follow once the economy returns to its natural path; therefore, the long-term rates do not spike as much as the current short-term rates. This leads to the narrowing of the spread between long and short-term rates. Hence, the yield curve begins to flatten. The higher short run rate would undermine investment activities in the economy and reduce output growth with a lag of one to two years.

The alternative hypothesis claims that the causal variable that gives rise to the predictive ability of the slope of the yield curve is the expectation concerning the future course of monetary policy. This hypothesis is particularly appealing to the framework of price rigidity in the short run, but flexibility over the longer course of time. Assume that the market anticipates strong economic activity in the future. As we have discussed above in the policy framework, there would be an expectation of a monetary squeeze, which would place upward pressure on the real rate of interest and slow down output growth in the future. However, this also has the potential of decreasing the current nominal long-term rate of interest if the inflation premium is expected to decline by more than what the future real rate of interest is expected to increase, thus causing the slope of the yield curve to flatten or even invert. Again, the result is that the higher nominal long-term rate would undermine investment activities in the economy and reduce output growth with a lag of one to two years. However, economists have frequently debated the credibility of this hypothesis. They argue that this hypothesis is built upon a positive relationship between output growth and inflation but during the 1970s and the 1980s, the relationship has been negative. Therefore, it is severely limited in its scope for explaining the relationship between the yield spread and economic activity.

Both the scenarios depicted above reveal that a narrowing of the spread between the long and short-term rate is likely to be associated with future recessions. The first hypothesis claims that the predictive power of the slope arises from policy adjustments affecting the short end of the yield curve. On the other hand, the alternative hypothesis holds that the predictive power of the slope arises from policy adjustments affecting the long end of the yield curve. However, given the close relationship between interest rates of different maturities, it is very challenging to distinguish them apart econometrically.

3.6 Theoretical Issues Concerning Role of Monetary Policy

Following, we discuss some of the recent events that further our underlined hypothesis of the relationship between yield slope and economic activity and present an account of a growing disagreement amongst economists about the future role of yield curve as a predictor.

Interest Rate Conundrum

Based on conventional wisdom, as we have discussed above, we would expect long rates to follow the same direction as short rates. When short rates rise due to monetary tightening, long rates would also be expected to go up (but with a lower magnitude) and vice versa. However, in the spring of 2004, when the Fed Chairman, Alan Greenspan, raised the federal fund rate from 1 percent to 3.5 percent, the yield on the 10-year
Treasury bond actually fell below the level at the onset of policy tightening. In congressional testimony on February 16, 2005, Alan Greenspan termed this rather unorthodox behavior of the long rates as the “conundrum”. Several studies have been conducted to investigate into the causes of this event. Craine and Martin (2009) concluded that the conundrum might have been triggered by the tremendous rise in the foreign central banks’ holdings of US treasury bonds. Their findings suggest that the rise in demand reduced the long-term bond yields by 0.8%, lower than what would have been expected in the absence of increased foreign demand. This excessive demand negated the effect of Fed’s monetary tightening and put downward pressure on long run yields. An alternative description of what led to the conundrum was believed to be the disappearance of the inflation premium due to strong economic outlook ahead and a growing confidence over Feds monetary actions. Kim and Wright (2005) estimated a three factor affine model of the yield curve and found empirical evidence that the risk premium on ten year treasury bonds decreased by 0.8% from 2004 to 2005.

The continuously falling long rates eventually led to the narrowing of yield spread and consequently to its flattening. However, at that time there was widespread consensus amongst policy makers that the currently observed flat yield curve is not necessarily an indicator of a following recession. For example, Chairman Ben Bernanke made the following statement before the Economic Club of New York in 2006:

*Although macroeconomic forecasting is fraught with hazards, I would not interpret the currently very flat yield curve as indicating a significant economic slowdown to come, for several reasons. First, in previous episodes when an inverted yield curve was followed by recession, the level of interest rates was quite high, consistent with considerable financial restraint. This time, both short- and long-term interest rates – in nominal and real terms – are relatively low by historical standards. Second, as I have already discussed, to the extent that the flattening or inversion of the yield curve is the result of a smaller term premium, the implications for future economic activity are positive rather than negative.*

(Bernanke, B. (2006). *Speech before Economic Club of New York*)

As Figure A6 in Appendix A depicts, a narrowing of the spread was actually followed by the great financial crisis beginning in the year 2007 furthering our case that the slope of the yield curve actually does help in predicting recession. The spread narrowed despite strong GDP and employment figures. However, these healthy figures were the result of excessive credit growth and a real estate ‘boom’, which was destined to come to a collapse. Beckworth (2008) correctly states, “I believe this recovery is being built on a marshland of debt and the bond market is reflecting this weakness”.

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5 See Appendix A for a chart illustrating the yield of 10-year treasury notes vs. federal fund rate target to point out the conundrum effect. (Figure A 5)

6 See Appendix A for a chart illustrating the conundrum and its relevance for subsequent economic activity. (Figure A 6)
Feds Quantitative Easing and US Liquidity Trap

An economy may find itself in a liquidity trap when its monetary authority becomes impotent over its ability to adjust the economy by changing the interest rates. This is because the prevailing rate is within the zero range (Keynes, 1936). In the case of a liquidity trap, the central bank may be tempted to pursue other exotic ways of stimulating the economy. The United States seems to have found itself in such a state post the credit crunch of 2008. The Fed has exhausted its ability to stimulate the economy via lowering the federal fund rate since the rate already stands close to the zero range. Therefore, the Fed has conducted quantitative easing in which it engages in the purchase of long term treasury instruments from selected financial institutions to depress the long run rates. This is different from Fed’s regular open market operations where it is rather involved in the purchase of short-term maturity instruments as opposed to long-term ones. Insofar, the Fed has purchased over a trillion dollar worth of treasury securities over the course of two rounds of QE beginning in November 2008. The rationale is to pump liquidity into the system and depress long-term rates to make funding easier and cheaper for businesses.

However, quantitative easing is not free of its problems. Its critics claim that by depressing long-term rates, the Fed has effectively harmed savers who now earn less in interest payments. This particularly holds for retirees who practically live off the interest payments from their life time savings. Moreover, it could generate excessive inflation in the economy if it fails to promote investment activity. Last but not least, it could very well be the case that the newly printed money never circulates into the system as banks hesitate from extending credit based on bad past experiences.

Lately, there has been an increasing speculation that when the policy rate bottoms out and the central bank is involved in quantitative easing operations, the relationship between the yield curve slope and future economic activity may follow a reverse course of action. This is because as QE depresses long-term rates, the yield spread narrows down and this leads to the flattening of the yield curve. In the case where QE is successful in decreasing the long run rate, it will stimulate future growth by making credit cheaper. In a scenario such as this, we would expect a negative relationship between the yield spread and future economic activity as opposed to our earlier laid hypothesis of a positive relationship. This is evident from the Japanese experience of the 1990s when Japan found itself in a state of liquidity trap, and the Bank of Japan engaged in QE activity to enhance credit activity. The Japanese yield spread, which has historically been a reasonable indicator of economic trouble ahead, ceased to operate its role following the enactment of QE measures. The last three recessions in the country have been preceded by a positive yield spread, and the yield curve has not undergone inversion since 1991. This is an important aspect of our paper as this reflects a structural break in the pattern of yield curve relationships.

---

7 See Appendix A for a chart illustrating how the Japanese yield spread failed to predict recessions in Japan under the state of liquidity trap. (Figure A 7)
Another possible variable from which the yield curve generates its predictive power is the stance of fiscal policy. Fiscal policy is at the discretion of government, and its two primary tools are government spending and tax rates. The relevance of fiscal policy increases in an environment where monetary policy becomes powerless in stimulating the economy. Based on the IS-LM framework as demonstrated in the figure below, an expansionary fiscal policy intended to stimulate consumption and investment activity in the economy, would shift the IS curve to the right, but it could have the consequence of giving rise to short term interest rates due to the ‘crowding out’ effect. Increased short-term rates would also increase long run rates based on the expectation hypothesis. However, long run rates do not rise with the same magnitude because the market expects government to cut down on its spending in the future to bring economic activity within reasonable bounds which shifts the IS curve back. This analysis indicates that a flattening of the yield curve resulting from expansionary fiscal policy may be an indicator of a decline in future economic activity.

An alternative view of assessing the impact of fiscal policy on the yield curve is to look at the prevailing federal budget deficits. Generally, countries with soaring budget deficits tend to exhibit higher overall interest rates. In the case where deficit is grossly high, to the point where there is an increasing sentiment in the market of a likely default, the short-term rates will tend to be much higher than long run rates. This is because investors demand a high compensation for assuming a default risk, but the longer horizon bond yields will not rise as much because investors would hope that over the long run, the government would eventually be able to impose austerity packages to bring its budget in control. This analysis confirms our hypothesis of an inverted yield curve’s ability to predict exacerbating economic situation in times ahead.

Bear in mind that the above mechanism applies to rather fiscally challenged economies. For a more stable economy like the US, the interpretation would be different. In this case, if there is an expectation among market participant of a growing budget deficit in the foreseeable future, the market would demand an additional risk premium as a compensation for the risk that the government would need to issue further bond securities to make up for the budget gap. This would lead to long-term rates being higher than the short-term rates since the risk lies in future. After all, if an investor

---

8 Consider the case of Greece which had 45.88 yield on its three-year bond obligations and 21.14 yield on its 10-year bond obligations with a budget deficit accounting for 9.5% of GDP as of December 2011. (Bank of Greece)

9 Consider the yields on US that stood at 0.423 for three-month maturity treasury bills and 2.046 for 10-year Treasury bond as of 4th quarter, 2011. (OECD)
were to purchase a long term bond security in an economy whose deficit is expected to increase, he might be able to cash a higher interest rate five years from now because the supply of long-term bond securities would have increased. This is based on the bond price and yield relationship outlined in section 3.1. In short, the longer the maturity horizon, the greater is the risk of things going wrong. This analysis suggests that a steep yield curve may be an indicator of expected increases in budget deficits, which could have different impacts on the overall economic growth.
4 Methodology

We aim to structure an econometric model that helps us translate the spread between the long and short-run yields at present period into the probability of a recession in the future period. The key issues we need to address include our choice of long term and short-term bond yields, forecasting horizon, our choice of econometric model, and the respective threshold for the occurrence of a recession.

There is a whole array of different bond yields available in the market corresponding to each bond instrument providing us with an abundant number of possible spread combinations. The objective is to select the combination whose yield components have a long historical record, which in turns alleviates its capability in making economic sense. Naturally, we tend to use the Treasury yields that are far apart with respect to their maturities. The obvious choice for the long end of the yield curve appears to be the ten-year Treasury bond yield, which is consistently available over a long period. As for the short end of the yield curve, three-month Treasury bills appear to be a rational choice. A yield spread based on these two yields would mandate relatively fundamental changes in the state of economy to turn it negative. Another consideration is that it’s the absolute level of the spread that is relevant for forecasting recessions rather than the actual change in spread. Suppose we currently observe a very steep yield curve with 300 basis points spread and certain market factors drive the spread down by 50 basis points, this would hardly lead to any dramatic increase in the probability of a recession for the decrease is rather minute.

The quarterly averages of the yields are used instead of point in time yields, as the daily spreads in yield cannot really be taken as credible signals due to their temporary persistence. Furthermore, there is empirical support that point-in-time Treasury bill yields at the last day of the calendar month carry systematic biases (Park & Reinganum, 1986). The sample data on quarterly yields begins from 1964, Q:3 and ends in 2011,Q:4. The data has been retrieved from OECD’s Statistical Database. Our spread computations assume the following equation:

\[
\text{SPREAD} = R^L - R^S \quad (\text{Eq. II})
\]

where the \( R^L \) is the ten-year quarterly Treasury bond yield and \( R^S \) is the three-month quarterly Treasury bill yield.

Although there is no general consensus amongst economists on the exact lag period between changes in the term structure of interest rate and its consequent effects on economic activity, it can be argued that the most optimal lag is the four-quarter period. This adheres in a conformance manner with the approach of Estrella and Hardouvelis (1991).

The criterion of our recession assessment is the one outlined by the National Bureau of Economic Research (NBER) which defines a recession as “a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales” (NBER). From the NBER, we obtain the starting and ending dates of each of the recessions that have occurred in our study time frame. They are listed as follows:
NBER-dated Recessions

<table>
<thead>
<tr>
<th>from</th>
<th>till</th>
</tr>
</thead>
<tbody>
<tr>
<td>year</td>
<td>quarter</td>
</tr>
<tr>
<td>1969</td>
<td>4</td>
</tr>
<tr>
<td>1973</td>
<td>4</td>
</tr>
<tr>
<td>1980</td>
<td>1</td>
</tr>
<tr>
<td>1981</td>
<td>3</td>
</tr>
<tr>
<td>1990</td>
<td>3</td>
</tr>
<tr>
<td>2001</td>
<td>1</td>
</tr>
<tr>
<td>2007</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 1 – Dates of the NBER-dated recession (NBER)

Addressing our choice of econometric model, we use a non-linear, binary, probability regression model known as the ‘Probit Model’ as opposed to the ‘Standard Linear Regression’ approach. The Probit model in our case uses the cumulative normal distribution function to assign a probability value for the occurrence of a recession derived from the lagged value of the spread. The focus here lies on predicting future recessions and not on estimating quantitative measures of economic growth as would be case if we used a linear regression approach. The Probit approach helps address the question frequently faced by policy makers concerning the future state of economy. Additionally, the binary random variable also rids us of the issue of spurious accuracy that are often linked with quantitative estimates.

A point to be noted here is that the Probit model emphasizes on the likelihood of a recession rather than on its severity. However, based on our above theoretical discussion, the narrower is the spread between long and short run yield, the greater is the scale of a likely recession ahead.

Finally yet importantly, after constructing our model, it is of fundamental importance to compute the minimum level of interest rate differential that needs to be realized for the model to signal a recession. For that, we use the ‘Leave-One-Out Cross Validation’ technique followed by deriving the ‘Receiver Operating Characteristic’ curve. The optimum threshold corresponds to the point of best performance in the ROC space. *(The technicalities and the mechanics of the statistical methods used are discussed thoroughly in the subsequent chapter).*
5 Empirical Findings

In the subsequent section, we present our empirical findings along with our analysis.

5.1 The Model

In pursuance of quantifying the predictive power of the spread of the yield curve in probabilistic terms, we use the Probit Model. The Probit Model belongs to the family of qualitative response models whose dependent variable is a binary, or dichotomous — one that can only take two possible values.

Theoretically, the model can be explicated by the following linear relationship:

\[ y_{t+k}^* = \beta' X_t + \varepsilon_t \]  

(Eq.III)

where \( y_{t}^* \) is a latent variable (unobservable) for the occurrence of a recession at time \( t \), \( k \) is the number of quarters ahead (represents the forecasting horizon), \( \varepsilon_t \) is the normally distributed error term (white noise process), \( \beta \) is the vector of coefficients, and \( X_t \) is the vector of values of the independent variables (including the intercept). The observable recession indicator, \( R_t \), is defined as:

\[ R_t = \begin{cases} 1, & \text{if } y_{t}^* > 0 \\ 0, & \text{otherwise} \end{cases} \]

So, if \( y_{t}^* > 0 \), then \( R_t = 1 \), which in turn implies a recession.

Equivalently, we can write:

\[
\Pr [ R_t = 1 | \text{SPREAD}_{t-4} ] = \Pr ( y_{t}^* > 0 ) \\
= \Pr ( \beta' X_t + \varepsilon_t > 0 ) \\
= \Pr ( \varepsilon_t > - \beta' X_t ) \\
= \Pr ( \varepsilon_t < \beta' X_t ) \quad (\text{symmetric since the distribution is normal}) \\
= \Phi ( \beta' X_t )
\]

The estimated equation would then be:

\[
\Pr [ R_t = 1 | \text{SPREAD}_{t-4} ] = \Phi ( \beta' X_t )
\]  

(Eq.IV)

where essentially \( \Pr [ R_t = 1 ] \) corresponds to \( \Pr [ (\text{Existence of NBER-dated Recession}), =1] \)

Accordingly,

\[ R_t = \begin{cases} 1, & \text{if the economy is in a recession at quarter } t \text{ according to NBER} \\ 0, & \text{if the economy is not in a recession at quarter } t \text{ according to NBER} \end{cases} \]

*For further clarification on the Probit Model, see ‘Appendix B’.*
Expanding the estimated equation, we get:

\[ \Pr [ R_t = 1 \mid \text{SPREAD}_{t-4} ] = \Phi (\alpha + \beta \cdot \text{SPREAD}_{t-4}) \]  
(Eq.V)

where \( \Pr \) denotes the probability, \( R_t \) denotes the unity during those quarters that represent official recessions by the NBER, \( \text{SPREAD}_{t-4} \) indicates that the spread variable accounted for in our model is lagged by 4 quarters (note that one can either have the spread variable with a lag of \( k \) quarters or the \( R_t \) variable with a lead of \( k \) quarters), \( \Phi \) is the normal cumulative distribution function of the Normit Model corresponding to \( \epsilon_t \), \( \alpha \) and \( \beta \) are our parameters.

The log likelihood function of the model presented above is estimated as follows:

\[ \log L = \sum_{R_t=1} \log \Phi(\beta' X_t) + \sum_{R_t=0} \log \Phi(1 - \beta' X_t) \]  
(Eq.VI)

Further expanding the above equation leads to:

\[ \log L = \sum_{R_t=1} \log \Phi(\alpha + \beta \cdot \text{SPREAD}_{t-4}) + \sum_{R_t=0} \log \Phi(1 - \alpha - \beta \cdot \text{SPREAD}_{t-4}) \]  
(Eq.VII)

We then maximize the log-likelihood function with respect to the unknown parameters \( \alpha \) and \( \beta \) the over the sample period of quarterly frequency from 1965:3 through 2011:4. This leads to:

\[ \Pr [ R_t = 1 \mid \text{SPREAD}_{t-4} ] = \Phi (-0.79278 - 0.53187 \cdot \text{SPREAD}_{t-4}) \]  
(Eq.VIII)

Both parameters, \( \alpha \) and \( \beta \), estimated using MLE are significant at the 5% level with estimated standard errors of 0.12531 and 0.09179, respectively. This equation shows that as the spread variable increases, the probability of a recession 4 quarters ahead decreases. The intercept and the interest rate differential both have a statistically significant effect on the probability of recession. However, since the relationship is non-linear, it is rather difficult to quantify this effect.

For the regression summary of the coefficients, refer to ‘Appendix C’.

### 5.2 Probability of a Recession

Having estimated the equation using MLE, plugging in values for the spread variable will give the predicted probability of a recession for each quarter. The results are presented in the chart below:
Figure 7 – The variation of the predicted probability of a recession as a function of the spread, compared to actual NBER-dated recessions.

This chart shows the predicted probability of a recession at time t as a function of the spread at time t-4. The regressand, the probability of a recession, is a binary variable indicating the existence or non-existence of an NBER-dated recession. The regressor is the four-quarter lagged slope of the yield curve. The light grey vertical columns designate actual NBER-dated recessions in the U.S. economy. The predicted probability of a recession is based on quarterly sampled data of the period 1964:3 – 2011:4 and accordingly estimated as of 1965:3. Thus, the chart above presents the in sample fit and predictive power of our Probit Model.

A brief look at the chart displays results in line with the hypothesis that the yield curve is a good measure for economic activity, at least for within-sample forecasting. Scrutinizing the graph, one can observe that all that NBER-dated recession have been accompanied by a spike in the predicted probability of a recession. Otherwise, with a few exceptions, the curve remains at low levels. The peak that was spotted in 1966-1967 is not associated with a recession but rather with an economic slowdown that occurred in that period. Moreover, the two spikes that were observed in 1996 and 1999 are false alarms (model type I error).

5.3 Econometric Issues

In order to get a better sense on the model's performance on new data, we perform a Leave-one-out cross validation (LOOCV) on the model using 0.5 as the threshold magnitude for the occurrence of a recession. In other words, the model will recognize a recession only when the predicted probability of a recession breaks through the threshold line of equation $y = 0.5$. 
Note that the performance figure we obtain after performing cross validation will be lower than that if we do not use cross validation. We use cross validation to get a better sense of the model’s predictive power on new data.

*For further clarification about the LOOCV technique, refer to ‘Appendix D’.*

In furtherance of quantifying the performance of our model in a more specific manner than the regular measures of goodness of the fit (e.g. McFadden R-squared or pseudo R-squared ...), we will construct the Error Matrix from a Binary Classifier and consider the Receiver Operating Characteristic (ROC) curve.

Since this paper uses the Probit Model, in which the response is binary in nature, we are dealing with a ‘Binary Classification’ (also known as a: ‘Two-Class Prediction Problem’). A Binary Classifier brings about four possible outcomes: if both the prediction and the actual value are positive, then the outcome is a ‘True Positive’ (TP). If both the prediction and the actual value are negative, then the outcome is a ‘True Negative’ (TN). If the prediction is positive and the actual value is negative, then the outcome is a ‘False Positive’ (FP). Finally, if the prediction is negative and the actual value is positive, then the outcome is a ‘False Negative’ (FN).

In this paper, our binary classifier has the following four possible outcomes:

- **TP:** Number of times the model correctly predicted a recession
- **TN:** Number of times the model correctly predicted no recession
- **FP:** Number of times the model incorrectly predicted a recession
- **FN:** Number of times the model incorrectly predicted no recession

The results are generated on the CRAN package and are presented in the Error Matrix (also known as a 2x2 Contingency Table or a Confusion Matrix, in particular cases) after LOOCV using a 0.5 threshold as follows:

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P</strong></td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>25</td>
<td>146</td>
</tr>
</tbody>
</table>

*Figure 8 – 2X2 Error Matrix after LOOCV of model using 0.5 recession threshold*
The following derivations and measurements are computed and reported as follows:

- **Sensitivity or True Positive Rate (TPR)** = 0.2647 = 26.47%. This means that 26.47% of the positives (occurrence of a recession) were correctly captured by the model. Hence, a recall rate of 26.47%.
- **False Positive Rate (FPR)** = 0.0395 = 3.95%. This means that the fall out rate is 3.95%.
- **Specificity (SPC) or True Negative Rate (TNR)** = 0.9605 = 96.05%. This means that the proportion of the negatives (non-occurrence of a recession), which were correctly captured by the model account for 96.05%.
- **Accuracy (ACC)** = 0.8333 = 83.33%. This means that the proximity of the results predicted by the model to the actual true value of those results is 83.33%.

For further clarification about the Error Matrix, refer to ‘Appendix D’.

The gist from the measurements presented here above shows that our model is accurate and returns ‘correct’ results in about 83% of the times. This high accuracy leads us further to believe that the spread or the interest rate differential between the long-term assets and the short-term assets is indeed a good measure and an accurate predictor of economic activity. In other words, upon relying on the term spread to forecast economic activity, one would be right at 83% of the times.

### 5.4 Recession Threshold Optimization

The actual values versus the predicted values in the Error Matrix were computed on a 0.5 threshold value. This means that the model would only signal a recession when the predicted probability of a recession is at least 0.5 (50%). This threshold value is not necessarily the optimum threshold. Lowering the threshold line allows the model to predict more recessions when they exist thus increasing the number of true positives but inevitably also increasing the number of false positives. Similarly, rising up the threshold line undermines the model’s capability to predicting recessions when they exist thus decreasing both the true positives and the false positives. This dilemma reveals the existing trade-off between sensitivity and specificity.

Therefore, in order to find the optimum threshold with the highest possible true positive rate (TPR) and the lowest possible false positive rate (FPR) we will resort to the ROC (Receiver Operating Characteristic).

For further clarification about ROC, refer to ‘Appendix D’.

In order to avoid overfitting, we perform LOOCV again, cross validating not only the model, but also the method of choosing the optimal threshold. Without LOOCV, the model would be overfitted leading to an inflated performance measure simply because the model will always perform better on the data used to fit it than it will on new data. Two hundred and one possible thresholds between zero and one (increments of 0.005) are compared for each fold of the cross-validation. The threshold that yields the point in the ROC space the farthest distance from the random guess line is used to predict whether there is a recession for the observation left out.
Using the CRAN package, the Error Matrix is regenerated after LOOCV but using the optimized threshold this time as follows:

<table>
<thead>
<tr>
<th>ACTUAL VALUE</th>
<th>PREDICTED VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$n$</td>
</tr>
<tr>
<td>$p'$</td>
<td>25 (TP)</td>
</tr>
<tr>
<td>$n'$</td>
<td>9 (FN)</td>
</tr>
</tbody>
</table>

Figure 9 – 2X2 Error Matrix after LOOCV of model using optimized recession threshold

The following derivations and measurements are computed and reported as follows:

- Sensitivity or True Positive Rate (TPR) = $0.7353 = 73.53\%$. This means that the recall rate or the proportion of the positives (occurrence of a recession), which were correctly captured by the model account for $73.53\%$.
- False Positive Rate (FPR) = $0.125 = 12.5\%$. This means that the fall out rate is $12.5\%$.
- Specificity (SPC) or True Negative Rate (TNR) = $0.875 = 87.5\%$. This means that the proportion of the negatives (non-occurrence of a recession), which were correctly captured by the model account for $87.5\%$.
- Accuracy (ACC) = $0.8494 = 84.94\%$. This means that the proximity of the results predicted by the model to the actual true value of those results is $84.94\%$.

Compared to previous results, we can see that the sensitivity, the false positive rate, and the accuracy of the model increased by 47.06, 8.55, and 1.61 percentage points, respectively. On the other hand, the specificity of the model decreased by 8.55 percentage points. However, the performance measure, the distance to the no-discrimination line, has increased greatly. The results are displayed in the ROC space in figure 10:
The figure above graphs the ROC Curve of model performance using different thresholds. The algorithm fits the Probit model regressing recession on lagged spread, and then finds the threshold value that yields a point furthest from the random guess line on the ROC curve.

Cross validation of threshold optimization using LOOCV, can be summarized as follows:

- For each round of cross validation:
  1. The Probit model is fit using all the data except one observation, which is treated as the test set.
  2. The threshold is optimized.
  3. Predictions for probability of a recession are made for the test set, using the model from step 1. A recession is predicted if the probability is higher than the threshold found in step 2.
  4. The predictions for the test set are checked against whether there actually was a recession or not in those quarters.
  5. TP, TN, FP, and FN counts are computed and stored.
Steps 1-5 are repeated n times, so that by the end, every observation has been left out exactly one time. There will be a total of n TP counts, n TN counts, n FP counts, and n FN counts. These counts are then averaged for overall rates (presented in the Error Matrix in figure 9).

5.5 Statement of Results

Figure 11 – The variation of the predicted probability of a recession as a function of the spread, compared to actual NBER-dated recessions and observed at different thresholds.

In the predicted probability of a recession chart present here above, the 0.5 threshold line and the optimized threshold line have been introduced.

The optimized threshold as computed by R is 0.26. This means that the threshold that had the best performance measure is the line of equation $y = 0.26$. Hence, we can say that the model should signal the occurrence of a recession when the predicted probability of a recession is 26% or more. This value corresponds to a spread of -0.28. In other words, when the interest rate differential hits -0.28%, we should expect a recession in the US economy.

The table hereunder provides different values of the spread as a function of the probability of a recession (the spread is presented in percentage terms):
<table>
<thead>
<tr>
<th>Pr [Rt=1]</th>
<th>SPREAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>2,604</td>
</tr>
<tr>
<td>.10</td>
<td>1,232</td>
</tr>
<tr>
<td>.15</td>
<td>0,67</td>
</tr>
<tr>
<td>.20</td>
<td>0,224</td>
</tr>
<tr>
<td>.25</td>
<td>-0,158</td>
</tr>
<tr>
<td>.30</td>
<td>-0,502</td>
</tr>
<tr>
<td>.35</td>
<td>-0,821</td>
</tr>
<tr>
<td>.40</td>
<td>-1,123</td>
</tr>
<tr>
<td>.45</td>
<td>-1,416</td>
</tr>
<tr>
<td>.50</td>
<td>-1,703</td>
</tr>
<tr>
<td>.55</td>
<td>-1,991</td>
</tr>
<tr>
<td>.60</td>
<td>-2,284</td>
</tr>
<tr>
<td>.65</td>
<td>-2,586</td>
</tr>
<tr>
<td>.70</td>
<td>-2,905</td>
</tr>
<tr>
<td>.75</td>
<td>-3,248</td>
</tr>
<tr>
<td>.80</td>
<td>-3,631</td>
</tr>
<tr>
<td>.85</td>
<td>-4,078</td>
</tr>
<tr>
<td>.90</td>
<td>-4,639</td>
</tr>
<tr>
<td>.95</td>
<td>-5,471</td>
</tr>
</tbody>
</table>

Table 2 – The yield spread corresponding to different recession probabilities
6 Conclusion

After graphing the predicted probability of a recession against the actual NBER-dated recessions in the U.S. economy, we cross validate our model using the LOOCV technique in order to get a better sense on the model’s performance on new data. As the results after cross validation remain appealing, the robustness of the model on new data is thus reliable. In order to find the ‘best’ threshold for the model to signal a recession without overfitting the model, we perform cross validation for threshold optimization using the LOOCV technique; this time, cross validating not only the model but also the method of computing the optimal threshold. The true positive and false positive rates are plotted on an ROC space and the coordinates of the point closest to the perfect classifier are then identified by the code. Using backward algorithm, the code recursively retrieves the threshold that produced the respective coordinates by simply going in a reverse manner along the computational sequence. We identify the optimized threshold to be 0.26 which in turn corresponds to a yield spread of -0.28%. This suggests that once the predicted probability of a recession breaks through 0.26, or alternatively, once the interest rate differential breaks through -0.28%, we should expect a recession in the U.S economy.

The empirical evidence provides substantial support for the theoretical relationship discussed in Section 3. The yield slope tends to be very responsive to changes in monetary policy, which in turn affect the probability of a recession. A monetary squeeze leads to an increase in the three-month Treasury bill yields and the ten-year Treasury bond yields. This is consistent with what the expectation hypothesis of term structure claims. However, the increase in the ten-year Treasury yield is of a lower magnitude compared to the short-term increase in yield that is again in accordance with our monetary policy framework. As a result, the spread between long term and short-term yields narrows down, and in our case, this has proven to be a potent indicator of recessions ahead.

To sum up our main findings, we conclude that the yield spread is a reasonable indicator of economic activity; therefore, it should be factored into the decisions made by policy makers, investors and corporations. A better assessment of the future state of the economy may be carried out by crosschecking the results produced by our model with sophisticated macroeconomic models.

6.1 Further Research

Although our results are encouraging and in accordance with the underlying theory, we put forward a few considerations that could perhaps enhance upon the predictive ability of our model. Historically, a lot of the discussion has been centered on whether the yield curve’s predictive ability has been subject to structural breaks. Estrella, Rodrigues, and Schich (2003) conduct several tests to check for structural breaks in the relationship between yield spread and economic activity and conclude their findings with no significant breaks. However, there is a strong possibility of a break post the great financial crisis due to the Fed’s quantitative easing measures. It is therefore important to conduct stability tests taking recent events into account to test the credibility of the model for the future.
Another interesting area of research could be to formulate a dynamic Probit model as opposed to our static model and contrast the results to check whether our findings are consistent. Finally yet importantly, similar research could be conducted across developing countries where financial markets are largely regulated to see whether the predictive power of the yield curve holds in a regulated environment.
List of References


List of References


List of References


E-Sources:


List of References


Appendix

Appendix A

*Figures*

Figure A 1 – US Federal Debt as a Percentage of GDP
Source: http://www.usgovernmentspending.com/

Figure A 2 – US Federal Deficit as a Percentage of GDP
Source: http://www.usgovernmentspending.com/
Figure A 3 – Bond Fund Inflows against Equity Fund Inflows
Source: http://behaviouralinvesting.blogspot.se/

Figure A 4 – US Treasury Yield Term Spread and Recession Dates
Appendix

Figure A 5 – Yield of 10-year Treasury Notes vs. Federal Fund Rate Target
Source: https://mises.org/daily/1843

Figure A 6 – Treasury Yield Spread – The ‘Conundrum’ Effect
Source: http://macromarketmusings.blogspot.se/2008/10/interest-rate-conundrum-that-wasnt.html
Figure A 7 – Japanese Gov. Bond Yield Term Spread and Recession Dates
Source: http://www.boj.or.jp/en/
Appendix

Appendix B

The Probit Model

The Probit Model (also known as the Normit Model) belongs to the family of qualitative response models where the independent variable is qualitative in nature. It is a non-linear probability model in which the explained variable or the regressand is a dichotomous or binary variable. This means that the regressand can only take two values – 0 and 1. For instance, 1 (representing males) and 0 (representing females). The Probit Model is especially useful when the dependent variable can not be observed (latent unobservable variable). One of the several merits of the Probit Model is that it assumes normal distribution of the error term.

However, it is worth noting that the regressand does not necessarily have to be dichotomous or binary. It can for instance be trichotomous or quadichotomous. We refer to model then as a polychotomous (multiple categories) response model. For example, take the case of gender again (in the animal kingdom): one can be male, female, or both (hermaphrodite). That would be a trichotomous response model.

The Probit Model follows a normal cumulative distribution function where the dependent variable may virtually take on any value ranging from $-\infty$ to $+\infty$ and the probability would remain bounded in the set [0,1]. In comparison to the Logit Model that follows a logistic cumulative distribution, the Probit Model’s CDF is more asymptotic to the lower left limit and the upper right limit of the probability bounds.
\[
\Pi_i = P(Y=1 \mid X) = P(I_i^* \leq I_i) = P(Z_i \leq \beta_1 + \beta_2X_i) = \Phi(\beta_1 + \beta_2X_i) = \Phi(\beta'X_i)
\]

where:

- \( \Pi \) \( \rightarrow \) Probability
- \( P(Y=1 \mid X) \) \( \rightarrow \) Probability that an event Y occurs given X
- \( I_i \) \( \rightarrow \) Latent variable (unobservable)
- \( I_i^* \) \( \rightarrow \) Threshold or critical value for the unobservable latent variable (unobservable)
- \( Z_i \) \( \rightarrow \) Standardized normal variable (normally distributed variable with mean 0 and unit variance). \( Z \sim N(0, \sigma^2) \)
- \( \Phi \) \( \rightarrow \) Standard normal cumulative distribution function
- \( X_i \) \( \rightarrow \) Vector of values of the independent variables (including the intercept)
- \( \beta \) \( \rightarrow \) Vector of coefficients

Normal CDF:

\[
\Phi(\beta'X_t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta'X_t} e^{-z^2/2} \, dz
\]

Since the Probit Model is a non-linear model in nature, the estimation using OLS (ordinary least squares) is not possible. We revert to the method of Maximum Likelihood Estimation (MLE). This is done by maximizing the log likelihood function using iterative algorithms (Trial and error).

The log likelihood function is:

\[
\log L = \sum_{Xt=1} \log \Phi(\alpha + \beta^*X_t) + \sum_{Xt=0} \log \Phi(1 - \alpha - \beta^*X_t)
\]
Appendix C

Regression Output

Call:
glm(formula = recession ~ lagged.spread, family = binomial(link = "probit"),
     data = thesis.sub)

Deviance Residuals:

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>1Q</th>
<th>Median</th>
<th>3Q</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.2173</td>
<td>-0.5945</td>
<td>-0.3120</td>
<td>-0.1192</td>
<td>2.2382</td>
</tr>
</tbody>
</table>

| Coefficient Estimates | Standard Error | z value | Pr (>|z|) |
|-----------------------|----------------|---------|----------|
| (intercept)           | -0.79278       | 0.12531 | -6.327   | 2.50e-10*** |
| lagged.spread         | -0.53187       | 0.09179 | -5.794   | 6.87e-09*** |

Significance Codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

Null Deviance 176.93 on 185 d.f.
Residual Deviance 128.90 on 184 d.f.

AIC 132.90

Number of Fisher Scoring iterations: 6
## Appendix D

### Statistical Methods

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>True Positive (TP)</strong></td>
<td>equivalent with: <em>hit</em></td>
</tr>
<tr>
<td><strong>True Negative (TN)</strong></td>
<td>equivalent with: <em>correct rejection</em></td>
</tr>
<tr>
<td><strong>False Positive (FP)</strong></td>
<td>equivalent with: <em>false alarm, type I error</em></td>
</tr>
<tr>
<td><strong>False Negative (FN)</strong></td>
<td>equivalent with: <em>miss, type II error</em></td>
</tr>
<tr>
<td><strong>Sensitivity or True Positive Rate (TPR)</strong></td>
<td>equivalent with: <em>hit rate, recall</em></td>
</tr>
<tr>
<td></td>
<td>$TPR = TP/P = TP/(TP+FN)$</td>
</tr>
<tr>
<td><strong>Specificity (SPC) or True Negative Rate (TNR)</strong></td>
<td>$SPC = TNR = TN/N = TN/(FP+TN) = 1-FPR$</td>
</tr>
<tr>
<td><strong>False Positive Rate (FPR)</strong></td>
<td>equivalent with: <em>fall out</em></td>
</tr>
<tr>
<td></td>
<td>$FPR = FP/N = FP/(FP+TN) = 1 - TNR$</td>
</tr>
<tr>
<td><strong>Accuracy (ACC)</strong></td>
<td>$ACC = (TP+TN)/(P+N) = (TP+TN)/(TP+FN+FP+TN)$</td>
</tr>
<tr>
<td><strong>Positive Predictive Value (PPV)</strong></td>
<td>equivalent with: <em>precision</em></td>
</tr>
<tr>
<td></td>
<td>$PPV = TP/(TP+FP)$</td>
</tr>
<tr>
<td><strong>Negative Predictive Value (NPV)</strong></td>
<td>$NPV = TN/(TN+FN)$</td>
</tr>
<tr>
<td><strong>False Discovery Rate (FDR)</strong></td>
<td>$FDR = FP/ (FP+TP)$</td>
</tr>
<tr>
<td><strong>Matthews Correlation Coefficient (MCC)</strong></td>
<td>$MCC = (TP<em>TN - FP</em>FN)/ SQRT(PP'NN')$</td>
</tr>
<tr>
<td><strong>F1 Score</strong></td>
<td>$F1 = 2TP/(P+P') = 2TP/(2TP+FN+FP)$</td>
</tr>
</tbody>
</table>
**Error Matrix (2x2 Contingency Table)**

<table>
<thead>
<tr>
<th>Actual Value</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Predicted Outcome</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p'$</td>
<td>True positive</td>
<td>False Positive</td>
</tr>
<tr>
<td>$n'$</td>
<td>False Negative</td>
<td>True Negative</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TOTAL</th>
<th>$P$</th>
<th>$N$</th>
</tr>
</thead>
</table>

**Leave-one-out Cross Validation:**

Cross validation or rotational estimation, is used to get a better sense on a model's performance on new data.

The Leave-one-out Cross Validation (LOOCV) technique is one of several cross validation techniques such as: two-fold cross validation, repeated random sub-sampling validation and k-fold cross validation.

The LOOCV or the n-fold cross validation leaves out one observation of the data set as the test subset and retains the rest of the data as the training subset. The model is fitted leaving out one-nth of the data. Then, we see how well it predicts results for that one-nth. This process is repeated n times (where n is equal to the number of observations) leaving out a different one-nth of the data (test set) each time. The total n performance rates are then averaged to get a single performance measure.

**ROC Curve:**

The ROC Curve plots the true positive rate (TPR) against the false positive rate (FPR) for all the different thresholds of a binary classifier. Hence, it is a graphical representation of the sensitivity against one minus the specificity. The diagonal no-discrimination line ($y = x$), at which no inference can be made, divides the ROC space into two classification parts: better than random (upper division of the space) and worse
Appendix

than random (lower division of the space). A model that randomly guessed recession or no recession would be no-discrimination line.

The true positive rate (TPR) is defined as the proportion of the true positives (TP) to the total number of positives (P) and the false positive rate is the proportion of the false positives (FP) to the negatives (N). Accordingly, the ROC space represents the existing relative trade-off between the true positives (TP) and the false positives (FP) as they both will move in the same direction (more true positives implies more false positives and vice versa). In other words, the ROC depicts the trade-off between sensitivity and specificity. Ideally, the perfect point in an ROC space would be that of coordinates (0,1) at the top left corner of the ROC space for it locks up maximum sensitivity and specificity. The point of coordinates (0, 1) in the ROC space is the Perfect Classifier. The distance from the random guess line is a way to measure the performance of the model.