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PRICE DISCRIMINATION OF BUYERS WITH IDENTICAL PREFERENCES AND COLLUSION IN A MODEL OF ADVERTISING

AGOSTINO MANDUCHI

Abstract. A model of advertising and price distributions is investigated whereby each seller can contact different buyers, whose preferences are identical, with different probabilities. The model features a continuum of equilibria parametrized by the ratio of the buyers contacted by one seller - differing across “market segments” - and by the other sellers. In general, the sellers practice price discrimination across segments. More asymmetric equilibria correspond to higher volumes of transactions and higher expected transaction prices. This results in a lower expected utility for the buyers and higher expected profits; thus, identifying areas of influence can help the sellers to support collusion.

Keywords: Price Discrimination, Collusion, Advertising, Price Distributions

Journal of Economic Literature Classification Numbers: D43, D83, L11

1. Introduction

The “law of one price” - that all transactions in one good must take place at the same price - is not an empirical regularity; in fact, “dispersion [of the sellers’ asking prices] is ubiquitous even for homogeneous goods” (Stigler, [8], p. 213). A widely accepted formal account of price dispersion views it as the result of randomization of the sellers’ prices, in situations in which some buyers are not aware of all prices - see among others Burdett and Judd [3], Butters [4], McAfee [6], Robert and Stahl [7], and Varian [9]. The main result of the present note is that, in situations of this type, the sellers’ expected profits are higher at equilibria at which each seller reserves a more asymmetric treatment to buyers belonging to different “market segments.” Market segmentation can then allow the sellers to implement collusive agreements, essentially by limiting the buyers’ knowledge of the competing price offers.

The segments are identified by a payoff-irrelevant feature of the otherwise identical buyers. For each segment, each seller simultaneously chooses both a price and a probability that the respective buyers will receive an “ad” with a price offer, namely an “advertising intensity.” The model features a continuum of equilibria at which each buyer is more likely to receive ads from one of the sellers, the “leader” of the respective segment, than from any other seller; each seller plays the role of the leader in one of the segments. Each equilibrium is characterized by a specific value of the ratio between the fractions of the buyers contacted within each segment by each non-leader and by the leader, or “degree of symmetry.” A symmetric equilibrium, properly understood, also exists. To simplify exposition, equilibria with the same degree of symmetry that differ between them in the identities of the leaders of the different segments are regarded as *one* equilibrium. At the asymmetric equilibria, the prices advertised by the leaders and by the non-leaders are drawn from different probability distributions, and

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each seller practices price discrimination across segments - both *ex-ante* and also *ex-post*, with probability 1.

Lower degrees of symmetry correspond to lower probabilities for the leader of each segment to actually compete with other sellers, and therefore to higher expected revenues per buyer contacted; this is also a result of the “less aggressive” distributions governing the leader’s prices. Expected revenue equalization across segments, which must hold if the sellers are to be active in all segments, requires then each seller to adopt less aggressive price distributions also in the segments where she is not the leader. With convex advertising cost functions, higher expected revenues per buyer coincide with higher average advertising intensities and with larger fractions of the buyers contacted by at least one seller. More asymmetric equilibria are then associated with larger expected total surpluses, even if the increases in the advertising costs are taken into account. Due to the higher volume of transactions and to the increase in the expected transaction prices, the sellers’ expected revenues and profits are higher at the more asymmetric equilibria. The buyers’ expected utility is however lower at the more asymmetric equilibria, as from their point of view the effect of the higher expected transaction prices dominates that of the higher volume of transactions.

A model of advertising with a symmetric structure that admits equilibria with asymmetric strategies is studied in McAfee [6]. In that case, the population of the buyers is *not* partitioned into segments, and the sellers choose their advertising intensities *before* choosing the prices. In equilibrium, a (unique) leader contacts a larger fraction of the buyers and faces a higher expected profit than each one of her competitors. Results that are similar in many respects to those reported here also hold if the advertising intensities are chosen before the prices, if the sellers are allowed to discriminate across buyers.

Baye *et al.* [1] consider asymmetric pricing strategies in Varian’s model of price distributions [9], whereby the probabilities for the buyers to receive the ads sent by each seller are exogenous. In that case, the seller’s expected profits do not depend on whether the equilibrium is symmetric, or not. Asymmetric equilibria *cannot* exist in Robert and Stahl [7], whereby the buyers can independently elicit price offers from the sellers, at a cost. This point will be further discussed below.

The model is described in Section 2. Section 3 focuses on the equilibria and their welfare properties. Section 4 contains some concluding remarks. The proofs are in the Appendix.

2. The model

There are an integer number $m > 1$ of *ex-ante* identical sellers of a homogeneous good, indexed by i , and a $[0, 1]$ continuum of buyers with identical preferences. m is taken as given; a comment on some possible consequences of entry is made in Section 4. All agents are risk-neutral. The sellers face no capacity constraints, and can produce indivisible units of a homogeneous good at a constant cost, set equal to 0 without any further loss of generality.

Buyers can learn about the prices through the ads sent by the sellers. Each buyer is willing to purchase one unit of the good at the lowest price reported in the ads that she receives, as far as such price is not higher than her reservation price $v > 0$. The sellers announcing the lowest price satisfying this requirement can all be chosen with the same probability.

The population of the buyers is partitioned into the m segments $[0, \frac{1}{m}), \dots, [\frac{m-1}{m}, 1]$, indexed by s . For each s , each seller i simultaneously chooses a price p_i^s and an advertising intensity $b_i^s \in [0, 1]$ (super- and sub-scripts respectively denote segments and sellers). If the sellers could choose different strategies for each single buyer, rather than for each segment, the model would admit a broader array of equilibria; however, the equilibria of the present version of the model would still be included in such array.

Buyers cannot influence the probability of receiving ads from any given seller, and the receipts of ads sent by different sellers to each buyer are independent events, conditional on the sellers' advertising intensities in the buyer's segment.

An average advertising intensity $B_i = \frac{1}{m} \sum_{s=1}^m b_i^s$ is associated with a cost $A(B_i)$. A is twice continuously differentiable, strictly increasing and strictly convex on $[0, 1]$. I also assume $A(0) = 0$, $A'(0) < v$, and $A'(1) > v$; the first two assumptions rule out inactive equilibria, whereas the third assumption makes it possible to dispense with the complications associated with the possibility of symmetric corner solutions. I focus on Perfect Bayesian Equilibria (Fudenberg and Tirole, [5]), the same equilibrium concept considered in Robert and Stahl [7].

3. Equilibria and welfare

As a first step towards the characterization of the equilibrium strategy of any given seller i , I conjecture that the prices advertised are independently distributed across segments, and the advertising intensities are independent of the realizations of the prices. These conjectures will be validated in the remaining part of the analysis. The cumulative distribution of the price set by i in segment s is denoted by $F_i^s(p_i^s)$. The following Lemma summarizes Facts (1) - (4) in McAfee [6], to which the reader is referred for the proof.

Lemma 1. *If for each seller $i = 1, \dots, m$ and segment $s = 1, \dots, m$ the advertising intensity b_i^s is independent of the prices $p_i^1, \dots, p_i^m$, and $b_i^s > 0$ holds, then $F_i^s(p_i^s)$ is continuous over $[0, v)$. Furthermore, $F_i^s(p_i^s)$ can be discontinuous at $p_i^s = v$ for at most one seller and $F_i^s(v) = 1$ must hold for each seller i and segment s .*

By Lemma 1, the probability that an ad received by a buyer in s from seller i , reporting a price $p_i^s \in [0, v)$, will actually lead to a transaction, given the other sellers' advertising intensities and price distributions, can be written as $\prod_{j \neq i} (1 - b_j^s F_j^s(p_i^s))$. This is also true if $p_i^s = v$, if the distributions of the prices set by each seller $j \neq i$ are continuous over $[0, v]$; otherwise, it is easy to see that i will never set $p_i^s = v$. The statement of the sellers' problems can be simplified by introducing the sets $\Delta_i = (\delta_i^1 \otimes \dots \otimes \delta_i^m)$, $i = 1, \dots, m$, where for any $s = 1, \dots, m$, δ_i^s is defined as follows:

$$\delta_i^s = \begin{cases} [0, v] & \text{if for every } j \neq i, F_j^s(p_j^s) \text{ is continuous over } [0, v], \\ [0, v) & \text{otherwise.} \end{cases}$$

Each seller i then maximizes her expected profit

$$\Pi_i = \frac{1}{m} \sum_{s=1}^m \left(b_i^s p_i^s \prod_{j \neq i} (1 - b_j^s F_j^s(p_i^s)) \right) - A(B_i), \quad (1)$$

by choosing price distributions $\{F_i^1(p_i^1), \dots, F_i^m(p_i^m)\}$ over Δ_i and advertising intensities $\{b_i^1, \dots, b_i^m\} \in [0, 1]^m$. The distributions of the prices actually advertised must be continuous, except possibly at $p_i^s = v$, and the price distributions and advertising intensities of each seller $j \neq i$ are taken as given.

The advertising intensity chosen by the leader of each segment at the equilibrium characterized by the degree of symmetry ε is denoted by $b(\varepsilon)$. ε is such that the advertising intensity chosen by each non-leader is equal to $\varepsilon b(\varepsilon)$; therefore, $\varepsilon \leq 1$ holds, and $\varepsilon = 1$ identifies the symmetric equilibrium. This convention allows to deal with the possibility of equilibria at which the non-leaders' advertising intensities are equal to 0. I also set $B(\varepsilon) = \frac{1+(m-1)\varepsilon}{m} b(\varepsilon)$.

Theorem 1 characterizes the equilibria studied and the responses of the advertising intensities to changes in the degree of symmetry. A further result concerning the response of the expected transaction prices, the proof of which builds on that of Theorem 3, is separately stated as Theorem 2.

Theorem 1. *There exists an $\underline{\varepsilon} \in [0, 1)$ such that every $\varepsilon \in [\underline{\varepsilon}, 1]$ is associated with an equilibrium at which the advertising intensity chosen by the leader of each segment, $b(\varepsilon)$, is defined by*

$$(1 - \varepsilon b(\varepsilon))^{m-1} v = A'(B(\varepsilon)). \quad (2)$$

ii) The leaders' advertising intensity $b(\varepsilon)$, the non-leaders' advertising intensity $\varepsilon b(\varepsilon)$ and each seller's average advertising intensity $B(\varepsilon)$ are respectively decreasing, increasing and decreasing in the degree of symmetry ε .

Theorem 2. *The expected transaction price is decreasing in the degree of symmetry ε .*

Remark 1. *If the marginal cost of additional ads can be equal to the monopoly price even when only buyers within one segment are targeted, namely if $A'(\frac{1}{m}) \geq v$ holds, the lowest admissible degree of symmetry $\underline{\varepsilon}$ is equal to 0, and each segment can effectively be monopolized by its leader.*

Remark 2. *Eqn. (9) in the proof of Theorem 1 shows that, if $\varepsilon \in (0, 1)$, each seller practices ex-ante price discrimination across segments, and the probability distribution of the leaders' prices first-order stochastically dominates the probability distribution of the non-leaders' prices. This comparison is not meaningful at the monopoly equilibrium, at which the leaders' price distribution degenerates into a mass point at v , and the non-leaders' distributions are irrelevant - see the proof of Theorem 1.*

Stochastic dominance of the leaders' over the non-leaders' price distributions and advertising intensities that are independent of the prices drawn identify a situation analogous to that in McAfee [6]; in both cases, higher (expected) prices are associated with higher advertising intensities. On the other hand, at the equilibrium of the model by Robert and Stahl [7], the sellers drawing *lower* prices from the respective probability distributions are those choosing the higher advertising intensities. This difference is due to the fact that in [7], unlike in [6] and in the present note, the buyers are allowed to elicit price offers. In equilibrium, the buyers who actually take advantage of this opportunity are those who receive no unsolicited ads from the sellers, and their search consists of one random draw from the uniform distribution over the population of the sellers. Each seller is then guaranteed a "captive" market represented by a fraction $\frac{1}{m}$ of the searchers, which absorbs a larger share of the output of the sellers charging higher prices. The marginal revenue associated with additional ads sent is therefore always lower for such sellers, because a larger fraction of their ads is received by buyers that would ultimately be their customers anyway; with convex advertising cost functions, this translates into lower advertising intensities.

Theorem 3 characterizes the relative welfare properties of equilibria with different degrees of symmetry. $\tau(\varepsilon)$, $\pi(\varepsilon)$ and $\beta(\varepsilon)$ respectively denote the expected total surplus generated by the exchanges, net of the sellers' advertising costs, the expected profit for each seller and the expected utility for each buyer at the equilibrium associated with any given $\varepsilon \in [\underline{\varepsilon}, 1]$:

$$\tau(\varepsilon) = \left(1 - (1 - b(\varepsilon))(1 - \varepsilon b(\varepsilon))^{m-1}\right) v - mA(B(\varepsilon)), \quad (3)$$

$$\pi(\varepsilon) = B(\varepsilon) A'(B(\varepsilon)) - A(B(\varepsilon)), \quad (4)$$

$$\beta(\varepsilon) = \left(1 - (1 - b(\varepsilon))(1 - \varepsilon b(\varepsilon))^{m-1}\right) v - mB(\varepsilon) A'(B(\varepsilon)). \quad (5)$$

Notice that $B(\varepsilon) A'(B(\varepsilon))$ is used in (4) and in (5) to express the sellers' expected revenues (see the proof of Theorem 1).

Table 1. Differences between the values of the welfare variables considered in Theorem 3 at the equilibrium with the lowest degree of symmetry and at the symmetric equilibrium, for different numbers of sellers.

	$m = 5$	$m = 10$	$m = 20$
Diff. in the expected total surplus, $\frac{\tau(\underline{\varepsilon}) - \tau(1)}{\tau(1)}$	0.0006	0.0002	0.0001
Diff. in the expected profit, $\frac{\pi(\underline{\varepsilon}) - \pi(1)}{\pi(1)}$	0.5242	0.4234	0.3676
Diff. in the buyers' expected utility, $\frac{\beta(\underline{\varepsilon}) - \beta(1)}{\beta(1)}$	-0.0121	-0.0031	-0.0011

Theorem 3. *The expected total surplus $\tau(\varepsilon)$, the expected profit $\pi(\varepsilon)$ and the buyers' expected utility $\beta(\varepsilon)$ are respectively decreasing, decreasing and increasing in the degree of symmetry ε .*

Table 1 reports the differences between the expected total surplus, the expected profit and the buyers' expected utility at the equilibria with $\varepsilon = \underline{\varepsilon}$ and with $\varepsilon = 1$, in three examples with 5, 10 and 20 sellers. Each difference is expressed as a fraction of the value of the respective variable at the equilibrium with $\varepsilon = 1$. In each example, v is equal to 1 and the advertising cost function is $A(B_i) = \frac{0.001B_i}{1-B_i}$, the same function considered in Robert and Stahl [7]. The numerical routines, in *Mathematica*, are available from the Author.

The largest differences are observed in the case of the expected profit per seller. The differences in the expected total surplus are quite small, basically because a fraction of the buyers very close to one is served even at the symmetric equilibrium. The differences in the buyers' expected utility are also small, due to the relatively small differences in the total surplus and to the fact that the large "percentage" differences in the expected profit do not translate into large absolute differences. All differences are smaller for larger numbers of sellers. With larger values of v - such as those considered in [7], where $v > 10$ - the order of magnitude of the differences in the sellers' expected profits are approximately the same as with $v = 1$, whereas the differences in the other two variables are substantially smaller.

4. Concluding remarks

The multiple equilibria of the present model clash with the unique, symmetric equilibrium in Robert and Stahl [7], whereby the buyers can also learn about the prices by independently eliciting offers from the sellers. In that case, different strategies followed by the sellers would necessarily have a counterpart in different price distributions. This would affect both the buyers' search strategies and the sellers' expected profits, and ultimately force the sellers facing lower expected profits to revise their strategies.

One question that arises is then: Would the possibility for the buyers to search *always* cause the asymmetric equilibria identified here to disappear? While a full investigation of this issue is left as a topic for future research, it is already possible to say that the answer to the previous question must be "no," in general. In an economy admitting the monopoly equilibrium highlighted in Remark 1, if the search cost $c > 0$ were viewed as the cost of visiting for the first time a seller whose ad was not received, the buyers would never be willing to search if the price in each segment were always equal to v . If the buyers knew the sellers' actual price distributions, each seller could take the whole market without advertising and realize a profit equal to the price, by setting a price no larger than $v - c$ with probability 1. However, if $v - c \leq \frac{b(\underline{\varepsilon})}{m}v - A\left(\frac{b(\underline{\varepsilon})}{m}\right)$ holds, then charging v would indeed be a best-response for each seller. As this inequality

can always be verified, for suitably large values of c below the reservation cost v , the monopoly equilibrium can then exist even if the buyers are allowed to search.

In principle, the higher expected profits corresponding to more asymmetric equilibria could be wiped out if a sufficiently large number of sellers were allowed to enter the industry. This would *not* necessarily happen, however, in a version of the model including a stage in which each one of a “large” number of potential sellers must decide whether to pay a cost to become active, or not. The augmented model would in fact admit equilibria qualitatively similar to those analyzed in Benoit and Krishna [2] in the case of finitely repeated games. At these equilibria, the incumbents-to-be would threaten to choose the pricing and advertising strategies corresponding to the symmetric equilibrium of the post-entry game, under which the expected revenues of the sellers - including the “maverick” entrants - would be insufficient to cover the entry cost.

Appendix: Proofs

A.1: Proof of Theorem 1. Differentiability and convexity of A , together with $A(0) = 0$, imply that for any given seller i and prices $\{p_i^1, \dots, p_i^m\} \in \Delta_i$ - not necessarily in the support of i 's respective distributions - the first-order conditions for the problem of maximizing (1) subject to the constraint $\{b_i^1, \dots, b_i^m\} \in [0, 1]^m$ are both necessary and sufficient for an optimum.

Let then F_l and F_n denote the distributions of the prices advertised in any given segment by the leader and by each non-leader, with supports $L \subseteq [0, v]$ and $N \subseteq [0, v]$. If both $b(\varepsilon) \in (0, 1)$ and $\varepsilon > 0$ hold, the first-order conditions for each seller are

$$p(1 - \varepsilon b(\varepsilon) F_n(p))^{m-1} = A'(B(\varepsilon)), \quad (6)$$

$$p(1 - \varepsilon b(\varepsilon) F_n(p))^{m-2} (1 - b(\varepsilon) F_l(p)) = A'(B(\varepsilon)), \quad (7)$$

respectively in the segment in which the seller plays the leader's role and in the remaining segments, for prices $p \in L$ and $p \in N$. (6) and (7) imply that each seller's expected revenue must be equal to $B(\varepsilon) A'(B(\varepsilon))$. (1) then implies that, at any price vector which *does* belong to the support of the joint distribution of the prices, we must have $B(\varepsilon) A'(B(\varepsilon)) - A(B(\varepsilon)) = \pi$, where π is the highest expected profit achievable. As the actual prices advertised do not appear in this equation, the values of $b(\varepsilon)$ and ε corresponding to $B(\varepsilon)$ must be optimal for *any* price vector in the support of i 's joint distribution, and the independence assumption in Lemma 1 can therefore be satisfied. Furthermore, for any price $p > 0$ which belongs both to L and to N , (6) and (7) together necessarily imply $b(\varepsilon) F_l(p) = \varepsilon b(\varepsilon) F_n(p)$. Notice that only F_l may have a mass at v ; this follows from Lemma 1 and also, if $m = 2$, from the fact that if F_n had a mass at v , the following part of the argument would force $\varepsilon > 1$, at the asymmetric equilibria. If all prices in a suitably small left-neighborhood of v , except possibly v itself, are actually advertised in all segments, then setting $\hat{F}_i = \lim_{p \uparrow v} F_i(p)$, $i = l, n$, we can certainly write $\hat{F}_n = 1$, and therefore $b(\varepsilon) \hat{F}_l = \varepsilon b(\varepsilon) \hat{F}_n = \varepsilon b(\varepsilon)$, namely $\hat{F}_l = \varepsilon$, if $b(\varepsilon) > 0$. (2) can then be recovered by considering (6) at $p = v$. To show that (2) does admit economically relevant solutions in $b(\varepsilon)$, let us observe that at $\varepsilon = 1$, our assumptions about A' imply that for $b(1)$ (and $B(1)$) equal to 0 and to a suitably large $\bar{b} < 1$, comparison of the two sides of (2) yields $v > A'(0)$ and $(1 - \varepsilon \bar{b})^{m-1} v < A'(\bar{b})$, respectively. As the left- and the right-hand side are monotonically decreasing and monotonically increasing in $b(\varepsilon)$, respectively, and both continuous, there must therefore exist a unique $b(1) \in (0, 1)$ such that (2) is verified. Differentiation of (2) yields:

$$\frac{db(\varepsilon)}{d\varepsilon} = - \frac{b(\varepsilon) \left(1 + \frac{1}{mv} \left(\frac{1}{v} A'(B(\varepsilon)) \right)^{-\frac{m-2}{m-1}} A''(B(\varepsilon)) \right)}{\varepsilon + \frac{1+(m-1)\varepsilon}{(m-1)mv} \left(\frac{1}{v} A'(B(\varepsilon)) \right)^{-\frac{m-2}{m-1}} A''(B(\varepsilon))} < 0. \quad (8)$$

$b(\varepsilon)$ is then continuous and decreasing in ε , and there must exist a lowest admissible value of ε , $\underline{\varepsilon}$, such that either $b(\underline{\varepsilon}) = 1$ and-or $\underline{\varepsilon} = 0$ holds (the two equalities can simultaneously hold only for non-generic economies). It is readily verified that the distributions of the leader's and the non-leader's prices can be written as

$$F_i(p) = \begin{cases} 0, & \text{if } p \leq \underline{p}, \\ \frac{1}{b_i} \left(1 - \left(\frac{p}{\underline{p}} \right)^{\frac{1}{m-1}} \right), & \text{if } p \in (\underline{p}, v), \\ 1, & \text{if } p \geq v, \end{cases} \quad (9)$$

where $i = l, n$, $b_l = b(\varepsilon)$, $b_n = \varepsilon b(\varepsilon)$ and $\underline{p} = A'(B(\varepsilon))$. Therefore, $\underline{p} > 0$ holds, and all prices in $[\underline{p}, v)$ can actually be advertised, consistently with the assumptions made above.

To complete the proof, we must deal with the fact that $b(\varepsilon) \in (0, 1)$ and $\varepsilon > 0$ cannot both hold at $\varepsilon = \underline{\varepsilon}$. As far as $\underline{\varepsilon} > 0$ holds, it is easy to show, by using a limit argument exploiting the properties of $b(\varepsilon)$, that (6) and (7) must still hold at $\varepsilon = \underline{\varepsilon}$, together with the conclusions established above, even if $b(\underline{\varepsilon}) = 1$. If $\underline{\varepsilon} = 0$ holds, then at $\varepsilon = \underline{\varepsilon}$ each leader always advertises the monopoly price and (2) can be recovered from (6) at $p = v$, again by using a limit argument - notice that $p = v$ must belong to L for any $\varepsilon > 0$ - while (7) is replaced by an inequality. (The expected revenue for each seller must then still be equal to $B(\varepsilon) A'(B(\varepsilon))$, the expression used in (4) and (5), in Section 3.)

ii) $\frac{db(\varepsilon)}{d\varepsilon} < 0$ is established in passing in the proof of Part i). Furthermore,

$$\frac{d(\varepsilon b(\varepsilon))}{d\varepsilon} = \frac{\frac{b(\varepsilon)}{(m-1)mv} \left(\frac{1}{v} A'(B(\varepsilon)) \right)^{-\frac{m-2}{m-1}} A''(B(\varepsilon))}{\varepsilon + \frac{1+(m-1)\varepsilon}{(m-1)mv} \left(\frac{1}{v} A'(B(\varepsilon)) \right)^{-\frac{m-2}{m-1}} A''(B(\varepsilon))} > 0 \quad (10)$$

can be shown to hold by using the expression for $\frac{db(\varepsilon)}{d\varepsilon}$ in (8), and

$$\frac{d(B(\varepsilon))}{d\varepsilon} = - \frac{b(\varepsilon)}{m \left(\varepsilon + \frac{1+(m-1)\varepsilon}{(m-1)mv} \left(\frac{1}{v} A'(B(\varepsilon)) \right)^{-\frac{m-2}{m-1}} A''(B(\varepsilon)) \right)} < 0 \quad (11)$$

can be shown to hold by using the same expression and the expression for $\frac{d(\varepsilon b(\varepsilon))}{d\varepsilon}$ in (10).

A.2: Proof of Theorem 3. (The proof of Theorem 2, using results from the present proof, is reported below.) By differentiating the expected total surplus in (3) and using (2), after some algebra, we obtain

$$\frac{d\tau(\varepsilon)}{d\varepsilon} = A'(B(\varepsilon)) \left(- \frac{(m-1)(1-\varepsilon)b(\varepsilon)}{1-\varepsilon b(\varepsilon)} \right) \frac{d(\varepsilon b(\varepsilon))}{d\varepsilon}.$$

As $\frac{d(\varepsilon b(\varepsilon))}{d\varepsilon} > 0$ holds - as we know from the proof of Part ii) of Theorem 1 - we can then conclude that $\frac{d\tau(\varepsilon)}{d\varepsilon} < 0$ must also hold. Furthermore, by differentiating the expected profit in (4), it is immediate to verify that $\frac{d(B(\varepsilon))}{d\varepsilon} < 0$ - also established in the proof of Part ii) of Theorem 2 - and convexity of A imply $\frac{d\pi(\varepsilon)}{d\varepsilon} < 0$. Finally, by differentiating the buyers' expected utility in (5), and using again (2), together with the expressions for $\frac{d(\varepsilon b(\varepsilon))}{d\varepsilon}$ and $\frac{dB(\varepsilon)}{d\varepsilon}$ in (10) and (11), we obtain

$$\frac{d\beta(\varepsilon)}{d\varepsilon} = \frac{\varepsilon (b(\varepsilon))^2 A''(B(\varepsilon))}{\varepsilon + \frac{1+(m-1)\varepsilon}{(m-1)mv} \left(\frac{1}{v} A'(B(\varepsilon)) \right)^{-\frac{m-2}{m-1}} A''(B(\varepsilon))} > 0.$$

A.3: Proof of Theorem 2. $\frac{dB(\varepsilon)}{d\varepsilon} < 0$ and $\frac{d\tau(\varepsilon)}{d\varepsilon} < 0$, respectively established in the proofs of Theorems 1 and 3, imply

$$\begin{aligned} & \frac{d \left(1 - (1 - b(\varepsilon)) (1 - \varepsilon b(\varepsilon))^{m-1} \right)}{d\varepsilon} \\ &= \frac{1}{v} \left(\frac{d\tau(\varepsilon)}{d\varepsilon} + mA'(B(\varepsilon)) \frac{dB(\varepsilon)}{d\varepsilon} \right) < 0. \end{aligned}$$

By risk-neutrality, each buyer's expected utility, defined in (5), can also be expressed as

$$\beta(\varepsilon) = \left(1 - (1 - b(\varepsilon)) (1 - \varepsilon b(\varepsilon))^{m-1} \right) (v - P),$$

where P denote the expected transaction price. $\frac{d\beta(\varepsilon)}{d\varepsilon} > 0$, established in the proof of Theorem 3, implies then that P is necessarily decreasing in ε .

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- Email address:* Agostino.Manduchi@ju.se

Jönköping International Business School, Box 1026 SE-551 11 Jönköping, Sweden