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Beyond the borders of the local. How 'instructional products' from Learning study can be shared and enhance student learning

Ulla Runesson Kempe**, Anna Lövström* and Björn Hellqvist*

*School of Education and Communication, Jönköping University, Sweden;

** School of Education and Communication, Jönköping University, Sweden and
University of Witwatersrand, Johannesburg, South Africa.

Purpose: In this paper, we present how experiences gained from a theory informed lesson study – learning study – in regard to a specific learning goal can be shared and used by other teachers in new contexts.

Design: A group of teachers worked together in a cyclic, iterative process of planning, evaluating and revising teaching. The aim was to provide possibilities for grade 2 and 3 students to become familiar with negative numbers. The teacher group draw the conclusion that the students needed to be able to differentiate some aspects of negative numbers. The conjecture was put to the test in a follow-up study with five new teachers and eight classes. One lesson was taught based on the empirical findings in the learning study.

Findings: The results suggest that teachers' collaborative work have possibilities to produce knowledge about critical aspects of learning that can be communicated and adopted in new contexts. The teachers in the follow up study were able to make sense of the results from learning study and incorporate the critical aspects in their teaching in a way that enhanced students' learning.

Originality: It is demonstrated that teacher collaboration in learning study can create knowledge that goes beyond the border of the local context.

Keywords: Lesson study, sharing instructional products, negative numbers, variation theory, knowledge production.

Article classification: Research paper

Introduction

There are extensive reports on the effectiveness of lesson study for teachers' improvement of teaching skills, how they learn to reflect, on changes in motivation and capacity to improve instruction and the development of content and pedagogical content knowledge (e.g. Lewis, Perry and Hurd, 2009). Furthermore, it is reported that Lesson Study can promote the establishment of learning communities and teacher collaboration, a culture of mutual accountability, shared goals for instruction and a common language for analyzing instruction (e.g. Chichibu and Kihara 2013; Hunter and Back, 2011; Toshiya and Toshiyuki, 2013). To us, with these purposes, lesson study will be restricted to a model for

professional development only, not as a system that can generate new and relevant knowledge recognized as a legitimate knowledge source for professionals.

Hiebert and Morris (2011) take lesson study further when they promote it as a system for “the creation of shared instructional products that guide classroom teaching” (p. 5).

‘Instructional products’ should be designed with a specific learning goal in focus and detailed enough to guide classroom instruction. An instructional product is the current answer to common and shared problems on teaching and learning. It is tentative, changeable and thereby open to improvement. Therefore, such ‘local theories’ embedded in the instructional product must be communicated, shared and improved by other teachers in other contexts. In this way, they could be tested and verified under new and local conditions.

However, it has been questioned whether this is possible. In a review of practitioner research Enthoven and de Bruijn (2010) studied whether practitioner research conducted in professional learning communities (PLC) and communities of practice (CoP) can be recognized as a legitimate knowledge source that produces knowledge relevant beyond the borders of the specific context. They searched for mechanisms that enable this kind of knowledge production and found that mechanisms for producing knowledge relevant beyond the local school context lacked. Teachers’ collaboration seemed to be aiming at practical innovations and change through teacher learning, and not through evaluation of the innovations. Kullberg (2012) and Runesson and Gustafsson (2012) have, on the other hand, demonstrated that knowledge gained in a specific theory-informed lesson study – learning study – about how to promote learning possibilities for students can be valuable for other teachers and developed in new contexts. The knowledge product reported on in these studies are not concerning broader levels of pedagogy such as: interactions, materials, lesson structures or teaching arrangements, but descriptions of what is critical for a targeted object of learning. Enthoven and de Bruijn (ibid.) further found it hard to assess whether and in what ways teacher collaboration can contribute to students’ learning. In this paper we address their critique when we report on a study in Swedish primary schools with the aim to evaluate ‘knowledge products’ produced in teacher collaboration. Results from one learning study of how to enhance young students’ learning of negative numbers were communicated and used by new teachers in a new context. The research questions were:

- What knowledge of young students’ learning of negative numbers was gained in the learning study?
- Is such knowledge relevant beyond the border of the local context?

Our paper starts with describing learning study and principles from variation theory that frames the approach. Next, the learning study about negative numbers and the identified critical aspects are described, followed by a section on how these were communicated and implemented in eight new classes in a follow up study. Finally, the learning outcomes from the follow up study is presented and discussed.

Learning study

The theory-informed version of lesson study referred to above is learning study (Marton and Pang, 2003; Cheng and Lo, 2013). It shares features with lesson study, such as the collaboration among teachers and the iterative design of planning, implementing, observing and revising of the lesson. Whereas the theoretical framing of lesson study is rarely made explicit, in learning study the process is framed, and the teachers are guided by a theory of learning; commonly variation theory (Marton, 2015). Just as with lesson study, there are reports on the positive effects of Learning study on teachers' professional development (e.g. Kullberg, Runesson, Marton et al. 2016; Lo, Chik and Pang, 2006) and it has been demonstrated that learning study is an approach for constructing knowledge concerning the objects of learning as well as the teaching-learning relationship, also.

It takes the professional task and learnings problems among specific groups as the point of departure and teachers' professional knowledge is a resource in the process. Learning study has been promoted as a research approach in analogy to 'clinical research' in medicine (Carlgren, 2012).

The knowledge produced in learning study is an instructional product, not in terms of a lesson plan or specific teaching methods, but what is found to be necessary to learn in order to develop a specific understanding, skill or attitude. Thus, it is not about learning in a general sense, but in relation to specific learning goals (cf. Hiebert and Morris, 2011). In learning study, variation theory serves as a tool for teachers to identify the necessary conditions of learning the object of learning. Learning, from this theoretical perspective, is seen as a change in one's way of experiencing something. How we experience something has to do with what aspects we notice and become aware of. For every object of learning there are certain critical aspects necessary to discern.

'Critical aspects' are dimensions of variation in the object of learning that the learner has not yet learned to discern and attend to. It has been suggested, however, that the critical aspects must be identified for every group of learners (Pang and Ki, 2016). Variation theory takes a relational perspective on learning, meaning that the critical aspects are not merely a feature of the content (a concept for instance), but a feature of the experienced object of learning. They cannot be derived at "from disciplinary knowledge alone or as taken-for-granted truths" (p. 333) Learners bring various experiences to the classroom and experience phenomena in different ways. Therefore, to identify the critical aspects, learners' ways of experiencing must be taken into account. In learning study, this is done by carefully diagnosing—via interviews and/or written tests—before and after the lesson. So, 'critical aspects' should be defined in relation to the phenomenon in question as experienced by learners rather than in relation to what is deemed critical in the curriculum or subject discipline (Pang and Ki, 2016, p. 328)

Marton (2015) asserts that one cannot become aware of new concepts or aspects without becoming aware of differences (i.e. variation). Variation theory is used when the teachers explore students' prior understanding and to what extent the object of learning has been achieved by the learners after instruction. The exploration of teaching and learning in the

learning study entails identifying what aspects of the object of learning that are critical for learning and how to make it possible for the learners to experience them. The process starts with assuming what might be critical aspects for learning. However, in the iteration, when analyzing student learning and what was taught, in terms of what was made possible to learn, in the lesson, hypothetical critical aspects might be rejected, new may be found, but mostly become specified.

When planning the lesson, variation theory is used for creating problems, example spaces and choosing representations, for example. From a variation theory perspective learning is a function of discernment, and discernment is a function of variation. We cannot discern a critical aspect of an object of learning if we do not experience some kind of variation, in relation to that aspect. Therefore, to make learning possible, certain patterns of variation must be manifested in the classroom, patterns that make simultaneous discernment of critical aspects possible (Marton, 2015; Marton and Booth, 1997; Marton and Tsui, 2004). Consequently, variation theory can be used for analysing the lesson also. Presuming that a presented variation in the midst of invariance can attend students' awareness to the targeted aspect, the lesson is studied from the point of view of what patterns of variation and what contrast are made to identify learning possibilities. When comparing two lessons arranged similarly and with the same topic taught, several studies have demonstrated that differences in the pattern of variation seems to have a significant role for student learning (e.g. Kullberg et al., 2014; Lo, 2012).

Teaching and learning negative numbers—some recommendations

Gaining understanding of the nature of negative numbers has been problematic for early mathematicians to comprehend (Bishop et al., 2014), as well as for teachers to teach and learners to learn (e.g. Ball, 1993). The difficulties have to do with the meaning of the numerical system and the magnitude and direction of the number, the meaning of arithmetic operations, and the meaning of the minus sign (Altıparmak, and Özdoğan, 2010). For Swedish students the meaning of the minus sign is probably particularly difficult, since in Swedish a number like -2 (in English: negative two) is pronounced as 'minus två' (minus two) and written -2 . Thus, there is no linguistic and symbolic difference between the minus as a sign for the operation and as a sign for the number (c.f. Gallardo, 1995).

It has been recommended that teaching of negative numbers should take the point of departure in real life problems or situations known from the children's experience and transformed into mathematical models. For instance, using 'a house' with floors above and below the ground floor, or a bird flying/diving above/below sea level, has been suggested (Ball, 1993). Negative numbers are not formally taught in the early grades in Sweden. The first encounter with negative numbers are usually contextualized within discussions about temperature below and above zero and with the help of the thermometer. However, there might be a risk with this. The number system and the ordering of integers might not be visible when negative numbers are talked about as 'minus-degrees' (in Swedish: 'minus-grader'). Every child probably knows that it is colder when the temperature is -10 degrees

C compared to a temperature of 3 degrees C. This may be confusing when they have to learn that -10 is a smaller number than 3. The limitations of metaphors for understanding and operating with negative numbers have been pointed out previously (e.g. Linchevski, and Williams, 1999). This was also found initially by the teachers in the learning study reported here. They found that when the metaphors thermometer and floors in a house were used for representing the abstract concept the students' learning did not improve. Therefore, the teachers decided to use the number line and numbers representing both as places and distances as a metaphor.

A Learning study on expanding students' number range from N->Z

In the learning study (LrS) one of the authors of this paper worked in collaboration with two primary school teachers and 64 students in four different classes in grade 2 and 3 (8–9 years old) in Sweden. The teachers wanted to extend the students' experience of numbers to include the negative numbers also. They had experienced that students in these grades thought that 0 was the smallest number or that they neglected the idea of a negative difference in, for instance, subtractions like $3 - 4 =$, and answered “zero” or “you can't calculate that”. From this, the aim of the learning study was formulated: *to find the critical aspects for realizing the existence of negative numbers*. In doing so, they explored what the students must learn in order to be familiar with integers and how to teach this in a way that would enhance the students' learning.

The learning study encompassed four cycles, that is, four lessons were taught with four different classes. Each class was taught one lesson (app. 50 mins.). Two of the teachers taught one lesson/class each (L1 and 2). Class/L3 and 4 was taught by the third teacher. The teacher group was introduced to learning study and variation theory in a one-day seminar before the first cycle. They met before and after each lesson in sessions where students' learning and the lessons were analyzed, and a new lesson was planned. In total the team had 10 meetings (app. 2 hours/meeting). One of the teachers (second author) was the team leader and took care of practicalities, recorded the lessons and prepared the meetings. Being appointed some time for the project she watched and transcribed the recordings (although not analyzing them in depth) them between the meetings.

The diagnostic test given before and after the lesson included four tasks where the biggest among five numbers should be identified. One task was to select among natural numbers only and in another among negative numbers only. Two of the tasks included integers (i.e. positive as well as negative numbers). Although, learning to operate with negative numbers was not the aim, one of the tasks included subtractions with negative difference, for instance $4 - 6 =$. How this was solved the teachers thought would demonstrate whether the students realized that there are numbers ‘below zero’.

The analysis of the diagnostic pre- and post-test, together with a close analysis of the recordings of the lesson, gave insights into what might be critical for learning and how the content must be handled to promote learning. Thus, when the learners failed to learn that which was targeted, the teachers had to go deeply into the lesson and inquire how the

content was handled and whether it was made possible to learn that which was intended. This analysis became the basis for the planning of the following lesson in the cycle, which was taught by a new teacher, and to *new students*, and again the recorded lesson and the diagnostic post-test were analyzed. The iteration proceeded until all classes were taught. In this way hypotheses about the critical aspects were tested in class and successively scrutinized and revised. So, the critical aspects emerged as a result of trying them out in class and carefully analyzing students' learning outcomes and what was made possible to learn in the lesson. When it was found that the learning outcomes were not as expected, the teachers had to consider the possibilities for learning during the lesson and, by being guided by variation theory, discuss learning in terms of discernment.

It was found that students' performance from pre- to post-test were the highest after lesson 4. This result indicated that the lesson 4 provided the best learning possibilities. To get a deeper insight into what was different in this lesson compared to the previous ones, all data were revisited, mainly by the second author/teacher. This analysis was much more in depth and with a critical view on the first analysis by asking: 'Is there evidence in the data that supports that the critical aspects have been identified or are there other things that might be critical? Were the assumed critical aspects enacted in the lesson in a way that made them possible to discern? Were the critical aspects opened up as dimensions of variation, and thus made possible to discern?'

The re-analysis resulted in a specification of the assumed critical aspects and a report of the study (Lövström, 2015; Runesson Kempe, and Lövström, 2017). It was concluded that for these students to understand the existence of negative numbers, it seemed to be necessary to be able:

1. To differentiate the value of two negative numbers
2. To differentiate the function of the minuend versus the function of the subtrahend in a subtraction
3. To differentiate the minus sign for negative numbers versus the minus sign for subtraction

To get the students to discern the critical aspects, the teacher group had constructed examples, based on the idea of variation/in-variance. The re-analysis demonstrated that the examples taken in the lessons were of importance. The choice and character of the example space (Watson and Mason, 2006) was changed and developed during the process. For instance, it was not until lesson 4 that the two examples $2 - 4 =$ and $-2 - 4 =$ thus, where the variation between the examples (c.f. Watson and Mason 2006, Ekdahl et al. 2016) are of a specific kind, were implemented and compared in the lesson. The lesson analysis demonstrated that when the examples were designed in a way that made a contrast, (e.g. between the double meaning of the minus-sign) possible to notice, the students performed better on the task $-2 - 2 = ?$ on the post-test compared to lessons where this contrast between examples were not present. The importance of how well the actual examples used fit the intended purposes has been pointed to previously (Kullberg, Runesson, and Mårtensson, 2014; Rowland, 2008). In this study, the particular examples and how they

were sequenced in terms of variation that opened up for possibilities to experience features of negative numbers, were identified by the teacher group.

Putting the conjectures to the test

Lövström (2015) concludes that when the critical aspect was phrased in terms of differentiation, that is what things could be compared, it indicated not just what dimension that must be opened up, but also what values in that dimension that were critical and needed to be contrasted (e.g. two or more negative numbers). Thus, critical aspects in terms of differentiation highlight a specific subject matter and students' experience of the content, and furthermore, provide directions for handling the content. To answer the research question; whether findings from learning study are relevant outside the borders of the local context, the conjectures of the nature of critical aspects and how to manifest them in the lesson were put to the test in a follow-up study.

Design of the follow up study

Eight classes of new learners (N=116) and five (partly) new teachers participated in the follow up study (hereafter called FS). All the teachers had more than 15 years of teaching experience. All, but one, were primary school teachers thus, not specialized in mathematics. Three of the teachers taught two classes each. One of them had participated in the learning study (LrS) and is one of the authors of this paper. All except one were, to a varying extent, familiar with variation theory. The teachers were selected based on previous interest in learning study and variation theory and asked to teach one lesson (three of the teachers in two different classes) about negative numbers. One of the classes was grade 7, a group of learners with difficulties in mathematics; all the other classes were grade 2 and 3. Swedish was the medium of instruction and the first language for the majority of the students, but several other languages were represented in all classes. The guardians had given their written consents to student participation. The students were given a test (with a few exceptions identical to the test in LrS) before and after the lesson.

The FS was planned in a 3-hour meeting with the teachers and two of the authors of this paper. Results from the pre-tests in the eight classes were presented and discussed and it was found that the 'new' group of students had similar problems to the students in LrS. So, the critical aspects identified in LrS were assumed to be valid for the new group of students also. What has been found to be critical for realizing the existence of negative numbers in the LrS: 1. To differentiate the value of two negative numbers 2. To differentiate the function of the minuend versus the function of the subtrahend in a subtraction 3. To differentiate the minus sign for negative numbers versus the minus sign for subtraction, was described and deeply discussed. How they had been manifested in the lesson was studied by observing the video-recording of lesson 4. Some sections were repeatedly paid attention to. It was specially observed and discussed in detail how the number line was used in the lesson. The aim was to conduct the eight lessons as similarly as possible in terms of how the three critical aspects were handled. Similar, it was important that all the examples presented and discussed in lesson 4, were present in all the 'following up-

lessons' just as the usage of the number line. Except for these requirements, the teachers were free to arrange the lesson in their own fashion; to choose group- or individual work, for example. All the lessons were video recorded.

The FS did not have the same iterative design as the LrS. It was conducted in parallel during the same week. The lesson was conducted mainly in whole class, intersected with individual and/or group-work. The interaction was more of a discussion between the teachers and the students with probing questions around the examples presented on the board. The examples used opened up dimensions of variation and were designed to make the critical aspects possible to discern. The teacher drew the learners' attention to differences in the midst of similarities and the students were required to justify their answers, sometimes after discussion in peers/groups. The lessons lasted about 60 minutes. In our experience, in Swedish schools it is uncommon that such a long period of time is allocated to whole class teaching of mathematics, at least among younger students. Still, the students seemed to remain concentrated and focused.

Data analysis

The data consists of video-recordings of eight lessons, and results from the pre- and post-tests (Appendix 1). Here, only results from task 1, 3, 4 and 9 are presented. With one exception (1d), the tasks were identical to the pre- and post-test in LrS. In task 1 (a–e), students should identify the biggest of five numbers. In a) all were positive numbers, b)-c) negative and positive numbers and zero, d) negative numbers and zero, and e) negative numbers only. The object of learning was not preliminary to operate with negative numbers, but in order to test if the students were able to experience that there are numbers < 0 , operations with negative difference were chosen. So, task 3 (8 items) involved subtractions with positive or negative difference. The subtrahend and the minuend were positive numbers except in g) where the subtrahend was negative. Similarly, task 4 was a subtraction with negative difference. Here the students should also give a justification of their answer. Task 9 was about the difference of the meaning of the minus sign. The test comprised another four tasks not accounted for here, however. The tasks were new to the main part of the students. Learners in grade 2 and 3 had not been formally taught negative numbers before. It is likely that the grade 7 students in the low ability group (12 of 116) had encountered integers before. However, this group did not perform better on the pre-test compared to their younger counterparts.

Results on the pre-test were compared to the post-test on each task and on a group level. A non-parametric sign test (Bailey, 1995; Vännman, 1990), which is independent of underlying probability distributions, has been practiced on the results.

Results

Preliminary results from the analysis of two tasks for all the groups are presented in Table 1 and 2.

[INSERT TABLE 1 HERE]

In task 1 there were learning gains in terms of numbers and percentage of students who displayed the targeted experience of integers on all except one item. As can be seen from Table 1, the frequency of correct answers was higher on all items on the post-test, except for a) (ordering positive numbers, which had a high rate from the beginning). The highest increase is on d): from 56% to 84% and e): from 29% to 73% who answered correctly. Item d) (ordering negative numbers and zero) and particularly e) (ordering negative numbers only) were initially more difficult than the others (lower scores on the pre-test compared to the other items). This could have been expected since this topic was new to the students. Therefore, we consider this as a significant improvement, although item d) and e) have lower scores on post-test compared to a-c. So, there were still students (15–26 %) who did not manage to find the biggest number among negative numbers or negative numbers and zero after the lesson.

The analysis of task 3 and 4 (Subtraction pos./neg. difference) suggests improvement on items except a) c) e) and f). This surprised us, since these subtractions (positive difference) are well known to the students. It might be that case that their encounter with subtractions with negative numbers have confused some students.

[INSERT TABLE 2 HERE]

The frequency of correct answers on the subtractions with negative difference is particularly interesting. Item b), d), g) and h) show a similar result. About half of the students could solve these correctly after being taught just one lesson. Before they were taught, the average frequency of correct answers on these items on the pre-test was slightly more than 10 % (13.6). So, there was a significant improvement on the post-test.

Item g) $-2 - 2 =$ is perhaps the most interesting. This item had the lowest frequency of correct answers before the lesson. Only 9% managed this on the pre-test. 44% answered correctly on the post-test. To be able to solve $-2 - 2 =$, one must differentiate the minus sign as an operation sign from the sign for negative numbers. The results indicate that almost half of the students could do that. This is supported by the result on task 9. This was designed to test whether the learners could understand the two meanings of the minus sign. On the pre-test, only 8 students (7%) could tell the difference between the minus sign in the operations $4 - 7 = -3$ and $4 - 3 = 1$ respectively. After the lesson, 42% answered correctly.

The non-parametric sign test, which is independent of underlying probability distributions, shows that students' gains in overall performance from pre- to post- test were statistically significant. The values of Tables 1 and 2 were added and the signs (positive or negative) of the differences between the number of students who solved the respectively items in the pre-test correctly and the corresponding number of students with correct solutions in the post-test were calculated. The pairs under items a in Table 1 and e respectively f in Table 2 were excepted, due to very small differences (less than 3 %) between the values. This leads to eleven pairs in the statistical test. A test parameter x was used, which represents the number of differences with negative signs. The introduced null hypothesis H_0 represents “no significant difference between the results in the pre- and post-tests”, while

the counter hypothesis H_1 assumes that there is a significant and positive impact on the post-test.

If H_0 is true, the test parameter x is binomial distributed, $\text{Bin}(11, 0.5)$. All conditions for x to be binomial distributed are fulfilled. Eleven independent attempts were made, one for each item, and the probability of negative (or positive) sign of the determined differences is 0.5. As can be observed the number of negative signs is two. $P(x \leq 2 \text{ when } H_0 \text{ is true}) = 0.0327$. This value is less than 0.05, which implies that H_0 can be rejected on the significance level of 5 %. In other words; the counter hypothesis is accepted, meaning that there is a significant positive impact on the post test.

Conclusions and discussion

The aim of the study was to assess whether a theory driven teacher collaboration – Learning study –can produce legitimate knowledge that is relevant beyond the border of the local context and has effect on students' learning (c.f. Enthoven and de Buijn, 2010). We have demonstrated that 'the knowledge product' of a learning study is not a description of the 'best' lesson design or an answer about how to teach a particular topic in general. It is a theoretically and empirically grounded description of some necessary conditions for learning, in this study about negative numbers among young students. The results from the learning study suggested that there were certain aspects that the learners in the study had to learn (i.e. are critical) for realizing the existence of negative numbers. To validate these findings the descriptions were communicated to a group of new teachers, who together planned for how to manifest them in a lesson with their students and under new conditions.

In this study we have not analyzed how the teaching was enacted, but the outcomes of the lesson in terms of students' learning gains. Drawing conclusions about the relation teaching and learning is of course problematic, since there is no direct cause effect between them. Still, our analysis shows improvement in students' learning in the FS, just as in LrS. That the identification of critical aspects and embedding them as patterns of variation in learning activities has effect on students' learning has been suggested previously (e.g. Cheung and Wong, 2014; Kullberg, 2012; Lo, 2012; Lo, Chick and Pang 2006). This study supports these findings, but indicates that they are relevant beyond the local context, also.

In this study we have tried to demonstrate that, it is possible that teachers' collaborative work can produce knowledge about critical aspects of learning that can be communicated and adopted in new contexts. The teachers in the FS were able to make sense of the results from LrS and incorporate the critical aspects in their teaching in a way that enhanced students' learning. This supports results of other studies (e.g. Lindström, 2017; Vikström, 20); if teachers are provided with tools that make them focus on the object of learning and the relationship between teaching and student learning, they can produce knowledge of a specific kind; critical aspects of the object of learning.

In their reviewed reports on teacher collaborative learning Enthoven and de Buijn (2010) points to a lack of explicit mechanisms contributing to knowledge production. Learning study and variation theory (Marton and Pang, 2013), we would suggest, can offer teachers

the external research resources and the more directive research program that Enthoven and de Bruijn (ibid.) think needs to be mobilized in order to create public knowledge in teachers' professional networks.

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