<http://www.diva-portal.org>

Postprint

This is the accepted version of a paper published in *International Journal for Lesson and Learning Studies*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

Mårtensson, P., Hansson, H. (2018)

Challenging teachers' ideas about what students need to learn: Teachers' collaborative work in subject didactic groups

International Journal for Lesson and Learning Studies, 7(2): 98-110

<https://doi.org/10.1108/IJLLS-11-2017-0048>

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:hj:diva-39201>

Abstract

Purpose

The purpose of this paper is to contribute to the understanding of the processes that make teachers learn in a collaborative arrangement similar to lesson study (LS) and learning study (LearS). The teachers in this collaboration wanted to enhance teaching and student learning (grades 4-7) about decimal numbers.

Design/methodology/approach

The analysis is based on data from five teachers' collaborative work in an adaptive arrangement of LS and LearS called subject didactic groups. Data consist of eight audio recordings of the teachers' meetings as well as written and photographic documentation of the meetings. The analysis was carried out through the lens of expansive learning within an activity system (Engeström, 1987). This entailed a focus on contradictions between teachers' ways of thinking and acting when individually and collaboratively developing their teaching, on the solutions to the conflicts produced by the teachers, and on how these challenged the teachers' ideas about what the students need to learn.

Findings

The authors identified contradictions between formative and summative assessment, exams and stressed students, prevailing norms about teaching and the theoretical tool used for planning and analyzing lessons and student learning, and the local curriculum and time constraints. The solutions to the conflicts were the driving force for developing new and more qualitative knowledge about what the students need to learn.

Originality/value

This paper gives explicit examples of contradictions and solutions that can challenge and drive teachers to expand their learning in an adaptive form of LS and LearS suited to daily teaching.

Keywords: collaboration, teachers' professional development, expansive learning, object of learning, lesson study, learning study, subject didactic groups

Introduction

The interest in different forms of professional learning communities (PLCs), in the context of education, has increased dramatically all over the world in recent decades. There can be a number of reasons for this, but the main argument put forward for PLC is that teacher participation in a PLC often positively affects both students' and teachers' learning (Fulton *et al.*, 2010; Vescio *et al.*, 2008). In the context of education, a PLC is regarded as a form of collegial arrangement in which teachers develop professional knowledge related to students learning (Fulton *et al.*, 2010). In research on PLC, it has been shown that teachers develop different kinds of professional knowledge (Fulton *et al.*, 2010; Robutti *et al.*, 2016; Vescio *et al.*, 2008).

According to Lo (2015), two types of PLCs, lesson study (LS) (Lewis and Tsuchida, 1998; Stigler and Hiebert, 1999) and learning study (LearS) (Pang and Marton, 2003), provide extra fertile environments for teachers' learning. However, what it is that affects teachers' learning in LS and LearS is rarely focused on in research on what teachers learn. This is something we also noticed when conducting a literature review of the *International Journal of Lesson and Learning Studies*. We did find a few studies that investigate what affects teachers' learning when participating in LS and LearS. For example, some conclude that social, cultural, and historical perspectives (Skott Krog and Møller, 2017) and feedback derived from theoretical assumptions (Huang and Han, 2015) affect learning in such collaborations. Shuilleabhain (2016) finds that teachers' experiences and their empirical observations and evaluations of research lessons affect teachers' learning. Wood *et al.* (2017) suggest that the key factors are the following: relating varying teaching designs to students' responses, sharing teaching experiences with group members, and overcoming constraints of the syllabus and assessment. In this paper, we focus on the learning process in terms of what it is that challenges and develops five teachers' learning about what the students need to learn in an adaptive arrangement of LearS called subject didactic group (SDG).

In LearS, the point of departure is the object of learning, which can be defined as an answer to the question "What is to be learned?" According to Marton (2015), an object of learning can be formulated in three different ways: as content, as

educational objectives, and as critical aspects. To refer to the content concerns learning of something, for example, a particular content taught in school such as equations. Marton (2015) elaborates an educational objective as a learning goal or a content-specific ability to be achieved after a teaching unit or a course, for example, solve a linear equation involving one unknown. Hence, an educational objective is the same for all students in a class. When formulating an object of learning in terms of critical aspects, one takes a specific student or group of students into consideration. Critical aspects are the aspects the students have not yet learned but need to learn to achieve the specific educational objective. Therefore, critical aspects cannot be determined in advance but must be discovered by teachers. This means that the object of learning in terms of critical aspects is dynamic and specific to a group of students in a certain situation at a certain time (Pang and Ki, 2016; Marton, 2015). Any given aspect is likely to be critical for some or several students, but it is unlikely that it is critical for all students (Marton, 2015). From a variation theory perspective, learning is becoming able to see things in a more powerful way by experiencing critical aspects. Furthermore, the theory states that which varies is most likely to be learned. To make such learning possible in teaching, the teacher must structure the critical aspects in terms of patterns of variation and invariance. These principles can guide teachers when planning and analyzing teaching and student learning.

Teachers' views of an object of learning may change during collaboration in LearS due to the iterative and systematic process of planning, conducting, observing, and analyzing lessons and student learning guided by variation theory. In some studies (e.g. Björkholm, 2015; Kullberg *et al.*, 2016; Mårtensson, 2015; Thorsten, 2015), the teachers took their own group of students' ways of seeing the object of learning into consideration and investigated the aspects the students focused on. This enhanced the ways the teachers transformed their knowledge, and it shows that the object of learning is relational in nature (Pang and Ki, 2016). Although we know that the iterative and systematic process and a focus on students' ways of seeing are important elements for teachers' knowledge about the object of learning, we do not know what contradictions teachers meet in their collaborative work or how those contradictions challenge teachers' ideas about what students need to learn. In this

paper, those questions will be investigated in the context of teachers' collaborative work in an SDG.

The case

Subject Didactic Groups

This study is conducted in the context of teachers' collaborative work in SDGs[1] in Sweden. The rationale for SDGs is to make collaborative efforts sustainable and integrated into teachers' everyday work, as this is not yet common in Sweden. SDG is an adaptive form of LearS with elements from teacher research groups (TRGs) (Paine and Ma, 1993). As in TRGs, teachers in SDGs are divided into groups by subject and grade, with one teacher assisting and facilitating the others. The focus of the groups is to develop teaching during weekly meetings. As in LearS, reflection on former lessons is an important element for planning new lessons. In SDGs, teachers focus on a long-term goal as well as on objects of learning for each lesson, whereas in LearS, the focus is only on a specific object of learning. This implies that there is a striking difference in the process of designing, enacting, and analyzing lessons. In LearS, the iterative process of revising and refining lessons is compulsory (Pang and Marton, 2003); furthermore, the lessons are commonly enacted in different classes. In a SDG, however, the teachers teach their own classes and link each lesson to the next based on reflections on the previous lesson. Due to the factors of time and other resources, in SDGs teachers normally do not observe the collaboratively planned lessons, which is an important element in LearS. On the other hand, all team members collect, during the jointly planned lessons, empirical data that are used in the evaluation and analysis of the lessons. In SDG and in LearS, teachers commonly use variation theory (Marton, 2015) as an explicit theory of learning to plan and analyze teaching and learning.

Context and data production

In 2014, SDGs were introduced as a pilot project in Sweden. Today more than 25 schools have implemented the arrangement. Our empirical case is taken from a compulsory comprehensive school (grades 1-9) in a small municipality in Sweden where SDGs were implemented in 2015. At that time, approximately half of the

teachers at the school were involved in the groups, and five teachers were facilitators. By the Autumn of 2016, when the study was conducted, the number of participating teachers and certified facilitators had increased, and the teachers were working in nine groups according to each teacher's main subject. All the teachers at the school had a given period in their time schedules allocated for collaboration.

In this paper, we draw on data collected in one mathematics group made up of five teachers. The teachers taught in grades 4-7, and their lengths of teaching experience ranged from 1.5 to 40 years. One of the teachers had undertaken a specific facilitator training course. In the role as a facilitator, she operated as a sort of a "specialist team member" doing the same things as the other teachers, but she also organized the meetings, led the discussions, motivated the group members, and asked challenging questions. For four months in 2016, the first author of this paper participated in the group, playing a mixed role of teacher and researcher. This meant that the author and the teachers worked toward the same objective – to enhance students' learning about decimal numbers. During five lesson-preparation meetings, the team mainly discussed the mathematical content, constructed tasks with the aim of mapping and analyzing students' understanding before teaching, and planned the first lesson. After planning the first lesson, the teachers conducted the lesson in their own classes and gave a written (pencil-and-paper) post-test to gain knowledge about student learning. The overall structure and the mathematical content of the different teachers' lessons were similar, but because the teachers taught mathematics in different grades, there were some differences in the lessons, such as the numbers used in instruction. After the first lesson, there were two post-lesson meetings during which the team analyzed the teaching and the tests. Based on that analysis and reflections about what to teach next, the group planned the second lesson. That lesson was also enacted by the teachers in their own classes, and a new post-lesson meeting was held afterward.

In the role of the researcher, the first author also served as a resource person (Pedretti and Hodson, 1995) and collected data in relation to the research questions in this study. Because there is always a risk of ambiguity with mixed roles and different roles among the participants in a group (Adamson and Walker, 2011), experiences, expectations, and the division of labor were continually discussed

during the study. The researcher taught mathematics education courses at the university and had much experience with different kinds of teacher collaboration as well as knowledge of the theoretical framework used, so the researcher held a lecture about variation theory for all the teachers and principals at the school. As in formative interventions (Engeström and Sannino, 2010), the researcher had no intention of controlling different variables or of leading the teachers in a certain direction. The autonomy of the teachers and the facilitator was the determining factor for every decision made. The discussions during the meetings were most often characterized by equality and mutual respect, even though there were disagreements and contradictions.

The key data were collected from the meetings and consists of eight audio recordings, which were transcribed. As our focus in this paper is the discussions between the teachers, written and photographic documents from the meetings were also collected. These included, for example, tasks from lessons and tests and teachers' notes about what they perceived to be important for students' learning.

Analysis

The primary unit of analysis is the SDG in mathematics, which we see here as an activity system (Engeström, 1987). Within an activity system, activities are mediated by tools and take place in a social setting. Figure 1 illustrates the general model of an activity system consisting of six nodes.

Figure 1

In our case, the subject is the group engaging and focusing on a joint object – to enhance students’ learning about decimal numbers. To achieve this, the group uses different kinds of tools, such as textbooks, tasks, teaching methods, pencil-and-paper tests, and variation theory. Additionally, rules, community, and division of labor are factors (three nodes in the bottom of the triangle) mediating and affecting sense and meaning within the activity system. In our example, rules are curriculum regulations, time, habits, and practice. Community comprises the people involved inside and outside school – parents and students, for example. Division of labor consists in the different roles among the participants – the teachers, a facilitator, and a researcher.

The analysis was based on the concept of expansive learning in formative interventions (Engeström and Sannino, 2010). This means that “learners learn something that is not yet there” (p. 2) by constructing a new concept or object for their activity. An important starting point in formative interventions is that neither the researcher nor the teachers know in advance what that concept or object will be. The analysis was carried out in three steps. Because learning is a result of questioning and contradictions within an activity system (Engeström, 1987), we first searched for and identified contradictions within the activity system. We focused on contradictions between the nodes as well as within the nodes. It is worth noting that contradictions need not be seen negatively but can be seen as inevitable and as the “driving force” for change and development that leads to the formation of a new expanded object (Engeström, 1987; Engeström and Sannino, 2010).

In the second step of analysis, we searched for “intermediate concepts” (Engeström, 2005), that is, new ideas and tools put forward by the group as solutions to the identified contradictions. In the third step, we focused more thoroughly on the meaning, or sense, and the outcome by drawing attention to how the object expanded in terms of the teachers’ ideas about what the students need to learn.

Result

Moving through the process in the SDG, learning took place and was manifested as changes in the object. These changes are presented below as ideas about what the students need to learn in terms of pieces of a topic, in terms of learning goals, and in

terms of critical aspects and tasks. Clusters of contradictions and questions raised within the activity were the fundamental driving force for that development. These clusters are presented in subheadings.

Idea 1: What the students need to learn, in terms of pieces of a topic

The teachers wanted to enhance teaching and student learning about decimal numbers. Compared to their ordinary work, the collaboration in the SDG enabled a change in focus that one teacher described as follows: “Instead of discussing how to teach, we are now discussing what to teach and what is to be learned.” This change implied that the teachers paid particular attention to common difficulties as well as to the pieces of the topic (decimal numbers) that they considered to be important for learning. An interesting feature of the discussions was that they tended to use phrases with similarities to the way the mathematical content is organized and presented in textbooks or to the mathematical understanding, skills, and knowledge students are supposed to acquire through different grades as expressed in the *Curriculum for the Compulsory School* (Skolverket, 2011) in Sweden[2]. The teachers’ opinion was that pieces such as the place-value structure of the number system, comparing and ordering decimals, and locating decimals on the number line were important. In attempts to organize the “thinking” and, furthermore, to promote and reach a strong consensus on which pieces of mathematics support student learning, the teachers constructed a mind map. Figure 2 illustrates the map, with the topic decimal numbers in the ellipse at the top and the network of pieces of mathematics below. The teachers grouped pieces of the topic together into three different themes or skills, which were organized in three columns: locate decimals on a number line, calculations, and compare and order decimals. The lines in the map indicate a relationship between the pieces and misconceptions: for example, the place-value structure related to zero is a placeholder.

Figure 2

From summative to formative assessment

Based on the pieces in the mind map, the teachers were going to design a pre-test in the style of a pencil-and-paper test to analyze and evaluate students' understandings and to identify areas of difficulties before planning the first lesson. Although formative and summative assessment processes form an important part of teachers' professional knowledge, tensions arose between those processes when designing and analyzing the test. The teachers had many experiences of formative assessment practice, such as giving ongoing information and feedback to students and parents about students' individual improvement, but they rarely made formative use of tests before teaching. On the contrary, they used tests and exams exclusively at the end of a mathematics course or after each chapter in the textbook to make summative assessments. Based on these experiences, and with respect to their students, some teachers found it wrong to assess students' understandings before the new instructional unit. One teacher said, "It is impossible for the students to show what they know if we haven't taught this in class before" and "I cannot see the point of students working on unfamiliar mathematical tasks [in pre-tests]; they will just get frustrated." Furthermore, it appeared that some students did not even try to answer all the questions and perform all the tasks in past pre-tests because they did not understand the aim of taking an exam before a course of instruction.

That opinion and the students' reactions were a starting point for identifying a contradiction between making formative and summative use of tests. The teachers started to question how they commonly use tests: "If we assess students' work at the end of the chapter, we can of course identify difficulties, but then we do not have much time to re-teach." The solution centered around the idea of problematizing the

meaning of what, that is, what the students need to learn. One teacher said, “We know that our students find subtraction much more difficult than addition. That’s why we are teaching it a lot, right? But what exactly do they struggle with?” The raising of the question made it possible for the teachers to think about all the pieces of mathematics summed up in the mind map, but in relation to their own students. Even though there was a change in focus, the teachers did not explicitly point out exactly what the students “struggled” with. But they were willing to explore student thinking and their conceptual understanding even more, so they constructed for the test several tasks in which the students were asked to express their mathematical reasoning.

From pre-tests to enquiry

After the students had taken the test, another contradiction was identified. The teachers had been told that a lot of students and some of their parents felt that the school used tests, diagnoses, and exams far too frequently. According to the parents, some of the children were overwhelmed and anxious. A prerequisite for a solution to the conflict between the students’ emotions and the use of a pre-test was that the teachers had gained new insights using tests formatively. Moreover, they realized that the students and their parents did not see the difference between formative and summative assessment processes. Based on that awareness, the teachers created a shared vision: “Next time, we have to emphasize and communicate that pre-tests are not used for [...] grading students.”

The teachers also discussed how to prepare a test with less formal characteristics so as to reduce students’ feelings of stress in the future. Solutions that came from that discussion were to tell the students not to write their names on the test and to give pre-tests on colored paper to clearly show the difference between an ordinary exam and a formative test. But the most crucial feature for dealing with the problem was that they started to see their daily activities as an enquiry in the classroom with opportunities to deepen their understanding of students’ understanding. Some ideas put forth with the intention to resolve the contradiction were expressed as follows: “We can have the same information from the diagnoses and exams that we commonly use, if we plan well” and “We can identify students’ understandings during the lessons if we observe and carefully listen to what they say.” Another idea

of how to collect feedback on students' learning to modify their teaching, but for future collaborations, was to use entry and/or exit notes[3].

Idea 2: What the students need to learn, in terms of learning goals

When analyzing the pre-test, the teachers started to count the number of correct and incorrect answers, which was the basis for identifying a lot of mistakes and errors associated with decimal numbers. More students than expected answered that “0.10 is greater than 0.9” and that “there are ten or no numbers between 5.2 and 5.3.” Furthermore, the students found it difficult to solve 17.7×2.5 , and several students answered 0.18 when adding $0.14 + 0.4$. By this time, the teachers had a general agreement about formative assessment and further enquiry, so they wanted to understand student errors in a more qualitative way. Consequently, they continued problematizing the meaning of what but now in relation to the incorrect answers on the test. When trying to identify what the students need to learn, the teachers realized that the topic decimal numbers was far too exhaustive to explore during their collaboration. For that reason, they divided the topic into two different learning goals on the basis of the age of the students: add decimals using a standard algorithm (grades 4 and 5) and multiply decimals using a standard algorithm and understand the difference between the algorithms for addition and multiplication (grades 6 and 7). Clearly, when discussing what the students need to learn, the teachers moved from a more peripheral understanding to a more central understanding in that the new learning goals were more distinct and clearly demarcated compared to the teachers' earlier conception. Another striking difference was that the new goals were not formulated based merely on the content. Rather, they depicted capabilities, or what the students are supposed to become able to do with the content. This change was crucial in that it made it possible for the teachers to expand their learning such that they re-evaluated their pre-existing idea about what the students need to learn. Some of the pieces of mathematics that they previously perceived to be essential and that they documented in the mind map were now considered less important – for example, “ordering decimals.” Other pieces were more thoroughly discussed in relation to teaching and students' understandings: for example, “The students know how to compare decimals, and they understand the place-value structure; it is something else they struggle with” and “Because they and we pronounce ‘four

tenths' [0.4] zero-point-four and 'fourteen hundredths' [0.14] zero-point-fourteen, they add 4 and 14 together." Even though the goals were more distinct and the pieces of the topic were re-evaluated and thoroughly discussed, the teachers still found it hard, when they went to plan their first lesson, to verbally express what aspects of the content their students need to learn to master the learning goal.

From "teaching wrong" to using errors in teaching

The teachers were supposed to use variation theory as a tool to design and analyze teaching and learning. However, although the teachers thought that it was easy to understand patterns of variation illustrated in the literature or in lectures they attended, they felt that applying the theory in practice was "most confusing." There may be several reasons for this, but the most striking feature of the problem was related to how variation theory challenged prevailing norms about teaching. From a variation theory perspective, it is of decisive importance for a teacher to identify the different ways students understand the same thing. Such knowledge usually implies that students' errors and incorrect answers are valuable information, not just for lesson planning but also for instruction. But some teachers had the contrary idea that errors and mistakes have a negative impact on learning and student achievement. One teacher expressed the contradiction between theory and that conviction: "Previously, we spent a lot of energy teaching the students how to count correctly; now we are going to teach them how to count incorrectly."

However, one of the teachers, invited to challenge the group through questions about their common teaching strategies, said, "We have told the students several times that 0.7 is greater than 0.35, and we have showed them why, right? Nevertheless, they make a lot of miscalculations, don't they?" and "Is it possible to try new things even if it is inconvenient?" Even though there were disagreements about teaching strategies, they adopted a common orientation when they once again focused on how to explore students' understanding. The new idea put forward as a solution to the contradiction was to not rely exclusively on tests and exit tickets to gain such knowledge but to continue the exploration more deeply during the lessons. One of the most common errors the students made on the pre-test was that they did not add the same place values together because they did not line up the decimal numbers correctly underneath each other in the standard addition algorithm. This mistake was

included in the first lesson plan, and the teachers decided to explore this more by asking probing questions and following up on students' responses. For example, they would say "What will happen if I write the digit 7 in 0.7 underneath the 5 in 0.35 when lining up the numbers?" "Why doesn't it work?" "How is this different from adding 7 to 35?" and "Please explain further!"

Idea 3: What the students need to learn, in terms of critical aspects and tasks

When the teachers discussed the lesson, they realized that most of their students performed well when using the standard algorithm for addition. Some of them even explained and described the step-by-step procedure when performing a task. However, the students found the algorithm for multiplication harder to use and understand. This was the starting point for an in-depth analysis of why the students in grades 4 and 5 gave answers such as 2.70, 13.10, 17, 26.7, or 39 when solving 13.5×2 and further, why students in grades 6 and 7 gave diverse answers when solving 1.3×4.5 . This prompted a threefold change. First, the teachers expressed more precisely for the first time what the students need to learn, even though there was a gradual change from an exclusive focus on pieces of the topic to students' thinking and conceptions about the mathematics during the process. The in-depth analysis revealed, for example, that some students treated integers as decimals and some students added instead of multiplying when using the standard algorithm for multiplication. Given this, three critical aspects were formulated in terms of distinctions (Mårtensson, 2015) the students and the teacher have to make to develop such learning: distinguish regrouping from multiplying integers, distinguish decimals from integers when they are lined up underneath each other, and distinguish multiplying from adding.

The second change relates to task design. Because the teachers were more explicit than before, they found it easier to construct tasks based on variation theory principles. For example, "find the errors" (Figure 3) was designed based on the identified critical aspects and ways to make them discernable. The task in Figure 3 was intended to be used in grades 4 and 5 (for grades 6 and 7, the decimal numbers 1.3×4.5 were used in a similar task). Example *a* in the task shows how to multiply 13.5 by 2 correctly. Example *b* is built on the critical aspect distinguish regrouping

from multiplying integers and shows a conception where the digit 5 in 13.5 is seen as an integer when multiplying 5 by 2. Furthermore, as there is a decimal sign in the algorithm, the answer 10 is documented as a separate entity and not regrouped. The critical aspect distinguish decimals from integers when they are lined up underneath each other were the basis for creating example *c*. In this example, 5 and 2 are seen as tenths and consequently there are two decimals in the answer. Example *d* relates to the critical aspect distinguish multiplying from adding. This example shows a common error: the students mix addition and multiplication together when operating.

Figure 3

The main ideas for lesson 2 were to start a whole-class discussion and reason about the different steps when solving the algorithm in example *a*. Next, the students were individually supposed to find the errors in examples *b* through *d* and then reason, in pairs and in class, about how an algorithm works. This implied a focus on explaining concepts such as place value, regrouping, and decimals. The new insights about what the students need to learn in terms of critical aspects and the ideas of how to make those critical aspects learnable undoubtedly led to the third change: further distinction of the learning goal. The teachers now expected their students to reason about the standard multiplication algorithm instead of add or multiply decimals using a standard algorithm, as was the case before.

From duplication of work to a combined practice

After lesson 2, a new contradiction emerged in the activity. When the school term began, the teachers expected their collaboration on decimal numbers to end after seven weeks, given that each topic in the local mathematics curriculum at the school was organized into such time units. Hence, the collaboration was considered to be an integrated part of their everyday work.

But as they moved through the process, they realized that it takes more than seven weeks to prepare, analyze, and evaluate pre-tests and lesson plans. Consequently, the

allocated time for the mathematics courses regulated in the local curriculum obstructed their intention to prepare more than two lessons during the collaboration. This meant that they had to plan lessons individually for their ordinary work and collaboratively in the SDG leading them to become overwhelmed and stressed. The expression of opinions such as “Using a magnifying glass as we do in this work is interesting and challenging, but we cannot afford it” and “It takes too much time; we must stick to the school curriculum” and “I think our work is duplicated” helped the teachers to focus on the root causes of the problem: the collaboration was not an integrated part of their daily work; instead, it was regarded as something extra, a luxury. In attempts to minimize the duplication of work, they started to discuss how to create conditions to combine collaboration with their ordinary work. The solution intended to resolve this contradiction was forward planning and a long-term perspective when choosing the next topic to explore. Instead of following the local curriculum strictly, the shared new vision was to plan and prepare (design and analyze tests and/or entry-tickets) the next topic three or four weeks earlier than before. The amendments involved in that vision would increase the opportunities to have the first lesson plan finished just as the new course or topic was supposed to begin according to the local curriculum.

Discussion

The teachers had only worked in the SDG for a year before participating in this study. They were also working inside Swedish culture, where teachers normally develop teaching individually (OECD, 2015). When they started to develop teaching collaboratively in a new systematic way, their ways of seeing and doing things were challenged substantially. The key to understand their ideas about what students need to learn and the challenges of their collaborative work in this study has been to view the SDG as an activity system in which not just tools mediate sense and meaning but also rules, community, and division of labor. Specific contradictions between and within the nodes were accumulated and detected in the activity. Contradictions between prevailing norms about teaching (rules) and tools have in this study been labeled as discussions about formative and summative assessment, and whether students’ mistakes are suitable for instruction. Furthermore, the contradiction between students and their parents (community) and tools was about exams and

stressed students, and the contradiction within rules of the activity was about the local curriculum and time constraints. However, the most important outcome here has nothing to do with contradictions but with agency – “participants’ ability and will to shape their activity system” (Engeström and Sannino, 2010, p. 20). As there was a willingness to deal with the contradictions and questionings raised in the group, it can be concluded that practical solutions – such as problematizing the meaning of what, focusing on student thinking and conceptual understanding, regarding daily activities as enquiry, and using probing questions and entry notes – were produced by the teachers themselves. The contradictions and the solutions helped the teachers expand their ideas from pre-existing or periphery ideas to more central or precise ideas about what the students need to learn, formulated and illustrated in terms of critical aspects within a mathematical task.

It can be interesting to reflect on the reasons for the willingness to deal with problems and diverse opinions in the group. One reason could be that the teachers in this study really wanted to learn from each other. However, we think that division of labor in terms of the different roles among the participants is an even more important aspect. The role of the facilitator cannot be overlooked. The facilitator in this study was an experienced teacher, but especially, she had knowledge about the variation theory tool and she had great experience of leading colleagues in different kinds of collaborations. Enthusing colleagues to try new things, asking effective questions to move the ideas forward, and focusing on the mathematical content are leadership qualities that seem to be important for the outcome.

Although this study illustrates only a single case, we think that these insights may be used as tools for qualitative change in other settings when trying to implement SDGs or similar PLCs in countries where systematic and collaborative teaching development is not yet a culture. It is most relevant, though, to assume that other challenges, contradictions, and tensions than those identified in this study will arise in different collaborative contexts. What is needed is an acceptance of the unexpected and an interest in dealing with the unexpected. In that way, the participants can acquire and apply new and more advanced solutions for their activities and for the future. The teachers in this study had a lot of discussions about how to summarize and document their work to enable new collaborations based on

their SDG or to enable other teachers to take part of their knowledge. Even though they captured and documented their knowledge on some occasions, in a mind map and a mathematical task, for example, the discussions largely became just visions. From an activity theory perspective, further contradictions between rules and tools could be an explanation. One such contradiction was expressed and dealt with toward the end of the collaboration. This appeared as resistance and feelings of stress because of a lack of time. We think, however, that documentation can be an important aspect of agency in teacher collaborations if knowledge is supposed to support the ability to act across contexts.

Comparing teachers' work in LearS and SDG, the use of variation theory is similar, but the systematic analysis of teaching and learning is somewhat different. To suit teachers' daily work, it is not compulsory in SDG to observe jointly planned lessons or to revise and refine lessons in an iterative cycle. On the other hand, in SDG, all teachers collect data from their own classes, which is then reflected upon and evaluated. Despite these differences, an interesting point, as we see it, is that SDG seems to be an arrangement that may help develop teachers' knowledge in ways similar to LearS (Björkholm, 2015; Kullberg *et al.*, 2016; Mårtensson, 2015; Pang and Ki, 2016; Thorsten, 2015). It is tempting to surmise that SDG may be as effective as LearS, if the purpose is to develop teachers' learning of what the students need to learn, even if teachers do not observe or revise lessons. After all, it seems that components such as challenges in relation to students' understanding, the theoretical framework, and time are more crucial. More studies on teachers' learning in SDG would be a reasonable way forward at this point.

The result can also be interesting in relation to variation theory. Marton (2015) describes three different meanings of the object of learning (what is to be learned), which can be formulated "in terms of content, in terms of educational objectives and in terms of critical aspects" (p. 22). These different ways illustrate levels of increasing precision in accordance with ideas 1-3, but here we have conceptualized the levels as developing and emerging knowledge driven by contradictions in a SDG. Worth highlighting, though, is that the development was not a one-way movement from idea 1 to idea 3 but a movement back and forth and sometimes parallel. Even if the result just illustrates a specific case, we see an

interesting difference between formulating the object of learning in terms of critical aspects and in terms of critical aspects and tasks (idea 3). As a critical aspect is a necessary aspect that the learner not has discerned and tells us what is expected to be learned to reach the learning goal (Marton, 2015), it highlights disciplinary knowledge in relation to learners' understandings. This may be considered effective, but critical aspects tell us less about how to make them learnable, thus less about how to teach. Our point is that, when teachers try to implement the new and more precise knowledge about critical aspects in practice, they are undoubtedly involved in constructing a new object for their activity. In this study, the object was formed into a task with the aim of making the critical aspects discernable; hence, the disciplinary knowledge was related to learners' understandings as well as to teaching. Such knowledge, we believe, has both a theoretical and a practical potential. We welcome a discussion about how to conceptualize the integration of features of teaching or acting with the concept of critical aspects.

Figures

Figure 1 The model of an activity system (Engeström, 1987, p. 78)

Figure 2 A mind-map over the topic decimal numbers showing what the students need to learn

a)

		1	3.	5	
	x			2	1
		2	7.	0	

b)

		1	3.	5	
	x			2	
		2	6.	1	0

c)

		1	3.	5	
	x			2	1
		2.	7	0	

d)

		1	3.	5	
	x			2	
		2	6.	7	

Figure 3 The teachers constructed the task “find the errors”, consisting of four examples build on three critical aspect

Notes

1. SDGs were developed by a group connected to Gothenburg University in Sweden with extensive experience of LearS as researchers, teacher educators, and teachers.
2. The Curriculum can be compared, for example, to the expectations in the *Principles and Standards for School Mathematics* in the USA.
3. A question or a task to answer passed out to the students at the start or at the end of a lesson that can be used as a formative assessment tool to provide teachers with a quick student diagnostic.

References

- Adamson, B. & Walker, E. (2011). Messy collaboration: Learning from a learning study. *Teaching and Teacher Education*, 27, 29-36.
- Björkholm, E. (2015). Unpacking the object of learning, *International Journal for Lesson and Learning Studies*, 4(3), 194-208.
- Engeström, Y. (1987). *Learning by Expanding: An activity theoretical approach to developmental research*. Helsinki: Orienta-Konsultit.
- Engeström, Y. (2005). *Developmental work research: expanding activity theory in practice*. Berlin: Lehmanns Media.
- Engeström, Y. & Sannino, A. (2010). Studies of expansive learning: Foundation, findings and future challenges'. *Educational Research Review*, 5, 1-24.
- Fulton, K., Doerr H., & Britton, T. (2010). *STEM teachers in professional learning communities: A knowledge synthesis*. Washington, D.C.: National Commission on Teaching & America's Future & West Ed.
- Huang, R. & Han, X. (2015). Developing mathematics teachers' competence through parallel lesson study. *International Journal for Lesson and Learning Studies*, 4(2), 100-117.
- Kullberg, A., Mårtensson, P., & Runesson, U. (2016). What is to be Learned? Teachers' Collective Inquiry into the Object of Learning, *Scandinavian Journal of Educational Research*, 60(3), 309-322.
- Lewis, C. & Tsuchida, I. (1999). A lesson is like a swiftly flowing river: research lessons and the improvement of Japanese education. *Improving schools*, 2(1), 48-56.
- Lo, M. L. (2015). Boundary crossing and learning from variation. *International Journal for Lesson and Learning Studies*, 4(2). Editorial.
- Marton, F. (2015). *Necessary conditions of learning*. New York: Routledge.
- Mårtensson, P. (2015). *Att få syn på avgörande skillnader: Lärares kunskap om lärandeobjektet (Learning to see distinctions: Teachers' gained knowledge of the object of learning)*. Doctoral thesis in didactics, no 29, Jönköping: School of Education and Communication, Jönköping University.
- OECD (2015). *Improving schools in Sweden: an OECD perspective*. Paris: OECD Publishing.
- Paine, L. & Ma, L. (1993). Teachers working together: A dialogue on organizational and cultural perspectives of Chinese teachers. *International Journal of Educational Research*, 19(8), 675-697.
- Pang, M. F. & Ki, W. W. (2016). Revisiting the idea of "Critical Aspects". *Scandinavian Journal of Educational Research*, 60(3), 323-336.

- Pang, M. F., & Marton, F. (2003). Beyond lesson study: Comparing two ways of facilitating the grasp of some economic concepts. *Instructional Science*, 31(3), 175-194.
- Pedretti, E. & Hodson, D. (1995). From rhetoric to action: Implementing STS education through action research. *Journal of Research in Science Teaching*, 32(5), 463-485.
- Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., Goos, M., Isoda, M., & Joubert, M. (2016). ICME international survey on teachers working and learning through collaboration: June 2016, *ZDM: The International Journal on Mathematics Education*, 48(5), 651-690.
- Shuilleabhain, A. N. (2016). Developing mathematics teachers' pedagogical content knowledge in lesson study. *International Journal for Lesson and Learning Studies*, 5(3), 12-226.
- Skolverket (2011). *The Curriculum for the compulsory school, preschool class and the recreation centre*. Stockholm: Skolverket.
- Skott Krog, C. & Møller, H. (2017). The individual teacher in lesson study collaboration, *International Journal for Lesson and Learning Studies*, 6(3), 216-232.
- Stigler, J. W., & Hiebert, J. (1999). *The Teaching Gap. Best ideas from the worlds teachers for improving education in the classroom*. New York: The Free Press.
- Thorsten, A. (2015). How teachers' practice knowledge is used and challenged in a Learning Study using Variation Theory as a tool. *International Journal for Lesson and Learning Studies*, 4(3), 274-287.
- Vescio, V., Ross, D., & Adams, A. (2008). A review of research on the impact of professional learning communities on teaching practice and student learning. *Teaching and Teacher Education*, 24, 80-91.
- Wood, K., Jaidin, H., Jawawi, R., Perera, J.S.H.Q., Salleh, S., Shahrill, M., & Sithamparam, S. (2017). How and what teachers learn from collaborative professional development. *International Journal for Lesson and Learning Studies*, 6(2), 151-168.